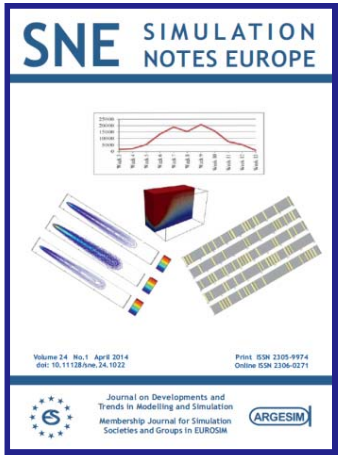
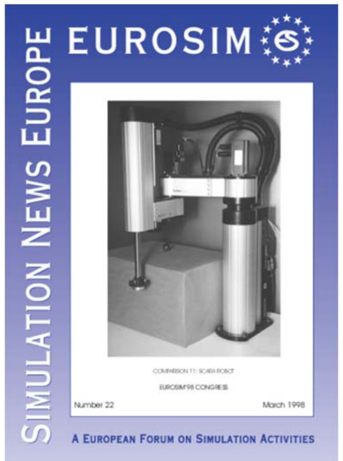
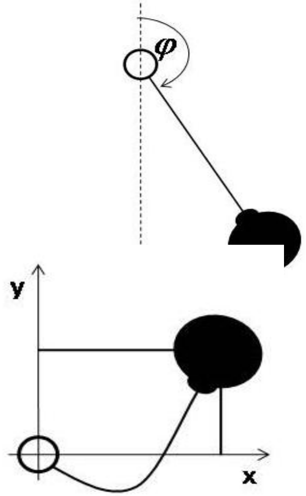


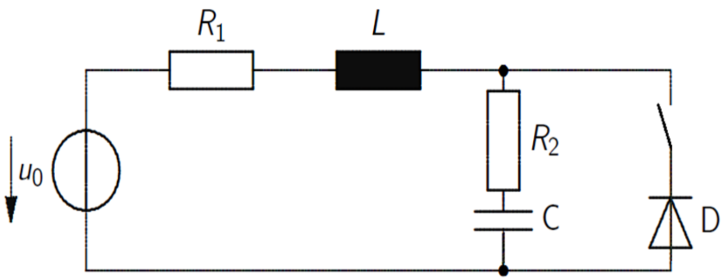


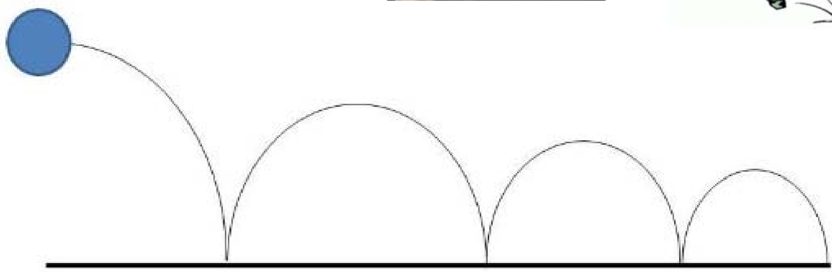
Comparison of Approaches for Modelling and Simulation of Structural-dynamic Systems – ARGESIM Benchmark C21 'State Events and Structural-dynamic Systems'



Andeas Körner, Felix Breitenecker

Mathematical Modelling and Simulation Group
Institute for Analysis and Scientific Computing, TU Wien, Austria





ARGESIM BENCHMARKS IN SNE

C1 Lithium-Cluster Dynamics, SNE 0(1), 1990

C2 Flexible Assembly System, SNE 1(1), 1991

C3 Generalized Class-E Amplifier, SNE 1(2), 1991

C4 Dining Philosophers I, SNE 1(3), 1991

C5 Two State Model, SNE 2(1), 1992

C6 Emergency Department SNE 2(3), 1992

C7 Constrained Pendulum, SNE 3(1), 1993

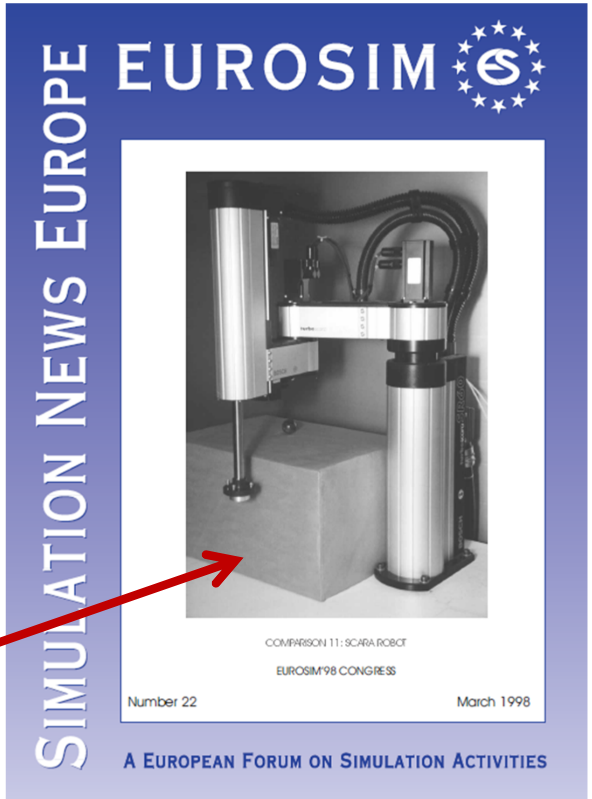
CP1 Parallel Simulation Techniques, SNE 4(1), 1994

C8 Canal-and-Lock System, SNE 6(1), 1996

C9 Fuzzy Control of a Two Tank System, SNE 6(2), 1996

C10 Dining Philosophers II, SNE 6(3), 1996

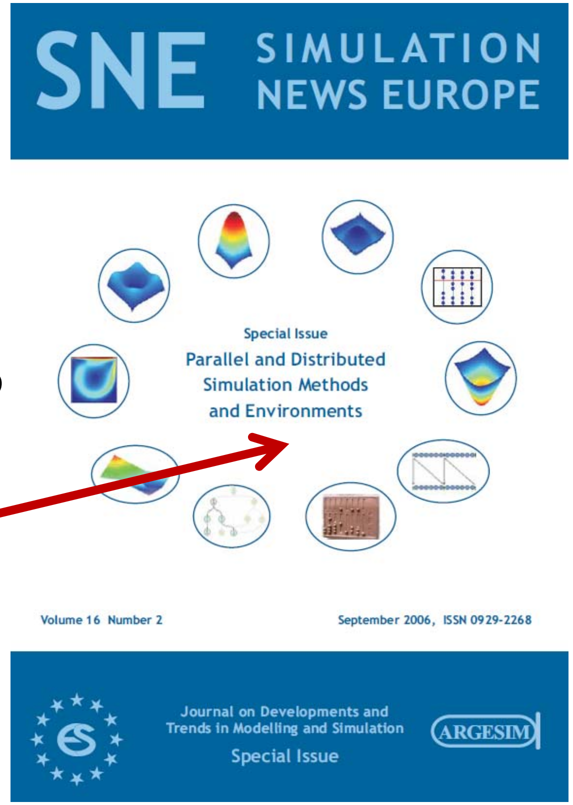
C11 SCARA Robot, SNE 8(1), 1998



Comparison of Simulation Software … … →

ARGESIM BENCHMARKS IN SNE

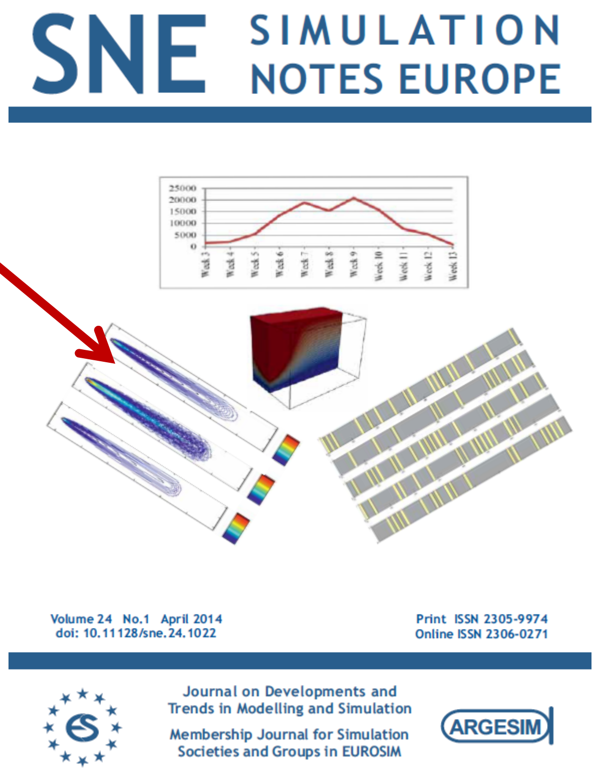
C12 Collision of Spheres, SNE 9(3), 1999
C13 Crane Crab and Embedded Control, SNE 11(1), 2001
C14 Supply Chain, SNE 11(2-3), 2001
C15 Clearance Identification, SNE 12(2-3), 2002
C16 Restaurant Business Dynamics, SNE 14(1), 2004
C17 Spatial Dynamics of SIR Epidemics, SNE 14(2-3), 2004;
C18 Neural Networks vs. Transfer Functions, SNE 15(1), 2005
C19 Pollution in Groundwater Flow, SNE 15(2-3), 2005
CP2 Parallel & Distributed Simulation, SNE 16(2), 2006



… → Benchmarks for Modelling Approaches and Simulation Implementations

ARGESIM BENCHMARKS IN SNE

C17 Spatial Dynamics of SIR Epidemics, rev. SNE 25(2), 2015
C18 Neural Networks vs. Transfer Functions, SNE 15(1), 2005
C19 Pollution in Groundwater Flow, rev. SNE 16(3-4), 2006
CP2 Parallel & Distributed Simulation, SNE 16(2), 2006
C20 Complex Assembly System, SNE 21(3-4), 2011



- SNE is publishing revised definitions
- Extended solution documentation (> 1 page)
- Extended Benchmarks: SNE introduces extended benchmarks, comparing modelling and simulation paradigms, or dealing with more complex models and experiments
- Documentation and publication in SNE of ‘solutions’ may take more pages – up to 10 pages SNE.

ARGESIM BENCHMARKS IN SNE

C17 Spatial Dynamics of SIR Epidemics, rev. SNE 25(2), 2015
C18 Neural Networks vs. Transfer Functions, SNE 15(1), 2005
C19 Pollution in Groundwater Flow, rev. SNE 16(3-4), 2006
CP2 Parallel & Distributed Simulation, SNE 16(2), 2006
C20 Complex Assembly System, SNE 21(3-4), 2011

**C21 State Events and Structural-dynamic Systems,
SNE 26(2), 2016**

*Benchmarks for Modelling Approaches
and Simulation Implementations*

ASC TU VIENNA – Mathematical Modelling and Simulation

DEVELOPMENT OF SYSTEM SIMULATION

$$\dot{\vec{x}}(t) = \dot{f}(t, \vec{x}(t), \vec{u}(t)), \qquad \vec{x}(t_0) = x_0,$$

ODE

$$\dot{\vec{x}}(t) = \vec{f}(t, \vec{x}(t), \vec{z}(t), \vec{u}(t), \vec{p}), \qquad \vec{x}(t_0) = x_0$$
$$\vec{g}(\vec{x}(t), \vec{z}(t), \vec{u}(t), \vec{p}) = \vec{0}$$

DAE

$$h^B(\vec{x}(t), \vec{u}(t), \vec{p}) \underset{\cdots}{\overset{!\ \pm}{=}} \vec{0} \overset{\hat{t}^B}{\Rightarrow} E^B(\vec{x}(\hat{t}^B), \vec{u}(\hat{t}^B), \vec{p})$$

State Events

DEVELOPMENT OF SYSTEM SIMULATION

$$\dot{\vec{x}}(t) = \vec{f}(t, \vec{x}(t), \vec{z}(t), \vec{u}(t), \vec{p}), \qquad \vec{x}(t_0) = x_0$$

$$\vec{g}(\vec{x}(t), \vec{z}(t), \vec{u}(t), \vec{p}) = \vec{0}$$

$$h^B(\vec{x}(t), \vec{u}(t), \vec{p}) \overset{! \pm}{\underset{\dots}{=}} \vec{0}^{\hat{t}^B} \Rightarrow E^B(\vec{x}(\hat{t}^B), \vec{u}(\hat{t}^B), \vec{p})$$

DAE

State Events

Event function

Event action

- Parameter change event – SE-P
- Input change event – SE-I
- State change event – SE-X
- Function change event – SE-F
- Structure change event – SE-S
- Output trace event – SE-O
- Algorithm event – SE-A

DEVELOPMENT OF SYSTEM SIMULATION

$$\dot{\vec{x}}(t) = \vec{f}(t, \vec{x}(t), \vec{z}(t), \vec{u}(t), \vec{p}), \qquad \vec{x}(t_0) = x_0$$

$$\vec{g}(\vec{x}(t), \vec{z}(t), \vec{u}(t), \vec{p}) = \vec{0}$$

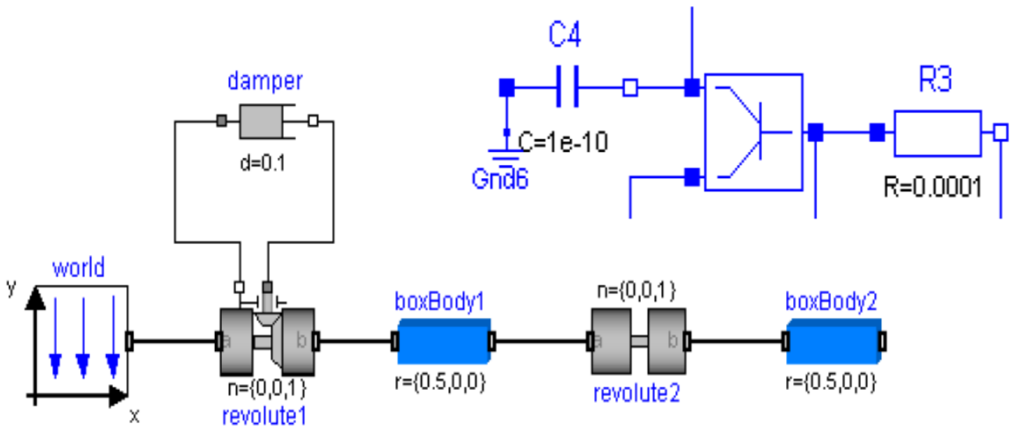
$$h^B(\vec{x}(t), \vec{u}(t), \vec{p}) \underset{\dots}{\overset{!\,\pm}{=}} \vec{0} \overset{\hat{t}^B}{\Rightarrow} E^B(\vec{x}(\hat{t}^B), \vec{u}(\hat{t}^B), \vec{p})$$

DAE

State Events

Event function

Event action



Physical Modelling –

State Space ‚unknown‘

STATE EVENT HANDLING

$$\dot{\vec{x}}(t) = \vec{f}(t, \vec{x}(t), \vec{z}(t), \vec{u}(t), \vec{p}), \qquad \vec{x}(t_0) = x_0$$

$$\vec{g}(\vec{x}(t), \vec{z}(t), \vec{u}(t), \vec{p}) = \vec{0}$$

DAE

$$h^B(\vec{x}(t), \vec{u}(t), \vec{p}) \overset{!\,\pm}{\underset{\dots}{=}} \vec{0} \overset{\hat{t}^B}{\Rightarrow} E^B(\vec{x}(\hat{t}^B), \vec{u}(\hat{t}^B), \vec{p})$$

State Events

- Parameter change event – SE-P
- Input change event – SE-I
- State change event – SE-X
- Function change event – SE-F
- Structure change event – SE-S
- Output trace event – SE-O
- Algorithm event – SE-A

Structural-dynamic Systems

STATE EVENT HANDLING

$$h^B(\vec{x}(t),\vec{u}(t),\vec{p}) \overset{!\ \pm}{\underset{\cdots}{=}} \vec{0} \overset{\hat{t}^B}{\Rightarrow} E^B(\vec{x}(\hat{t}^B),\vec{u}(\hat{t}^B),\vec{p})$$

State Events

Structural-dynamic Systems

- Parameter change event – SE-P
- Input change event – SE-I
- State change event – SE-X
- Function change event – SE-F
- Structure change event – SE-S
- Output trace event – SE-O
- Algorithm event – SE-A

1. *Detection* of the event
2. *Localisation* of event
3. **Event *Action***
4. *Restart* of solver

ASC TU VIENNA — Mathematical Modelling and Simulation

Structural-dynamic Systems

$$h^B(\vec{x}(t),\vec{u}(t),\vec{p}) \underset{\cdots}{\overset{!\ \pm}{=}} \vec{0} \overset{\hat{t}^B}{\Rightarrow} E^B(\vec{x}(\hat{t}^B),\vec{u}(\hat{t}^B),\vec{p})$$

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- Parameter change event –
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- State change event
- Function change event
- Structure change event
- Output trace event
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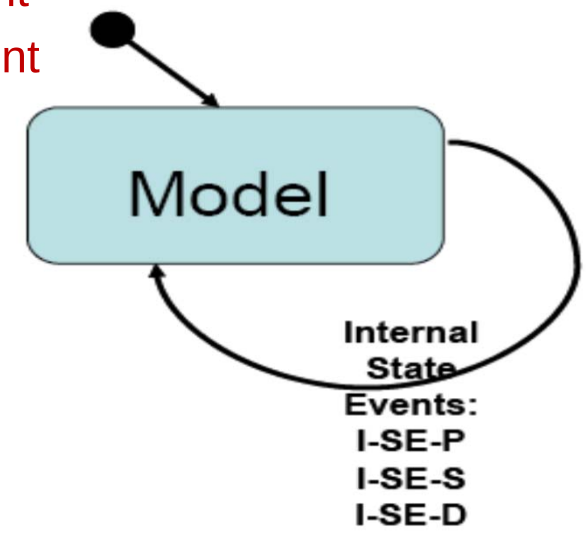

Structural-dynamic Systems

$$h^B(\vec{x}(t),\vec{u}(t),\vec{p}) \overset{!\,\pm}{\underset{\dots}{=}} \vec{0} \xrightarrow{\hat{t}^B} E^B(\vec{x}(\hat{t}^B),\vec{u}(\hat{t}^B),\vec{p})$$

1. *Detection* of the event
2. *Localisation* of event
3. **Event *Action***
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- Parameter change event –
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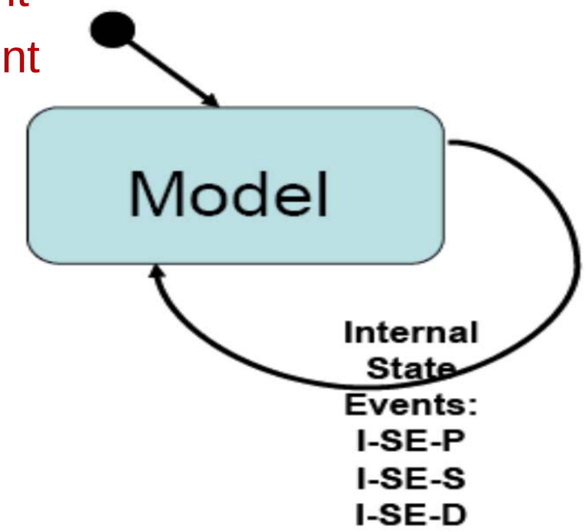
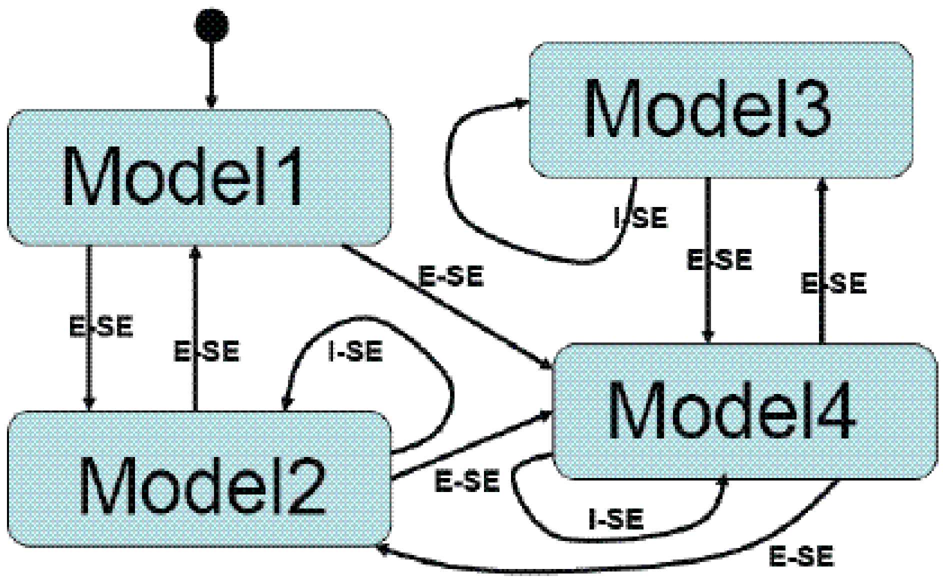
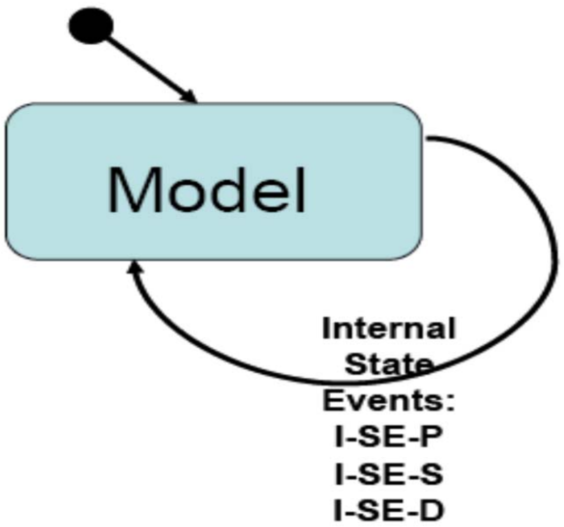


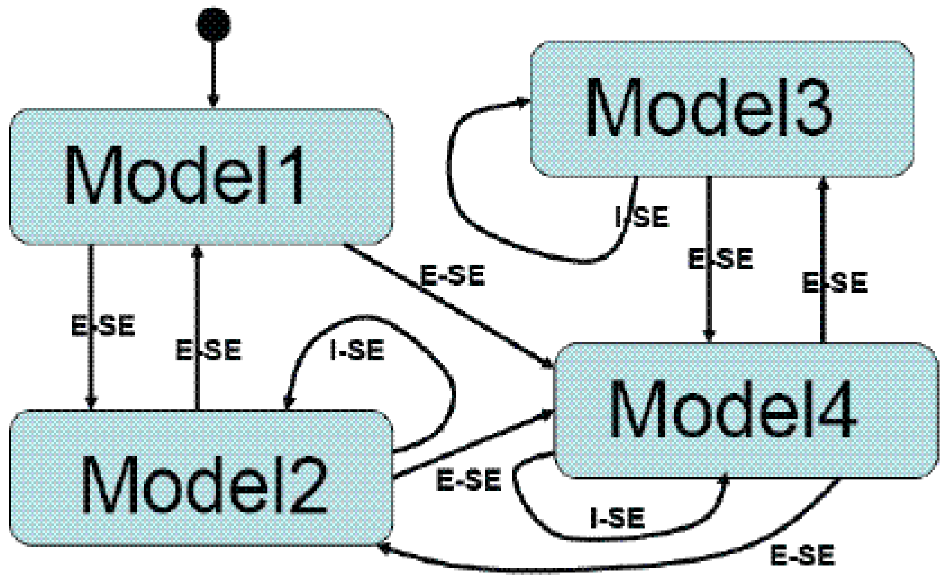


ASC TU VIENNA Mathematical Modelling and Simulation

Benchmark Structural-dynamic Systems

3 Case Studies

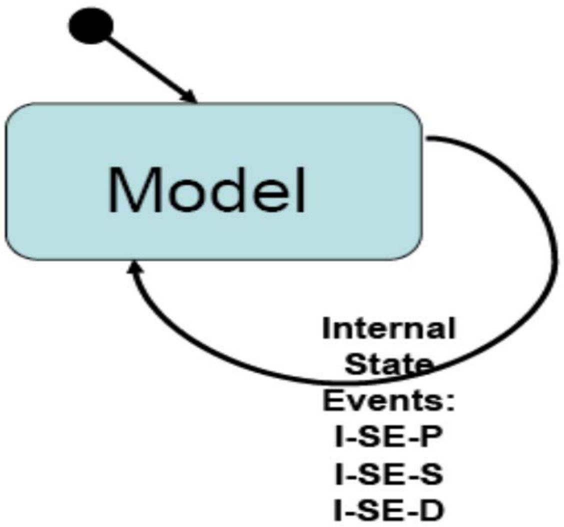


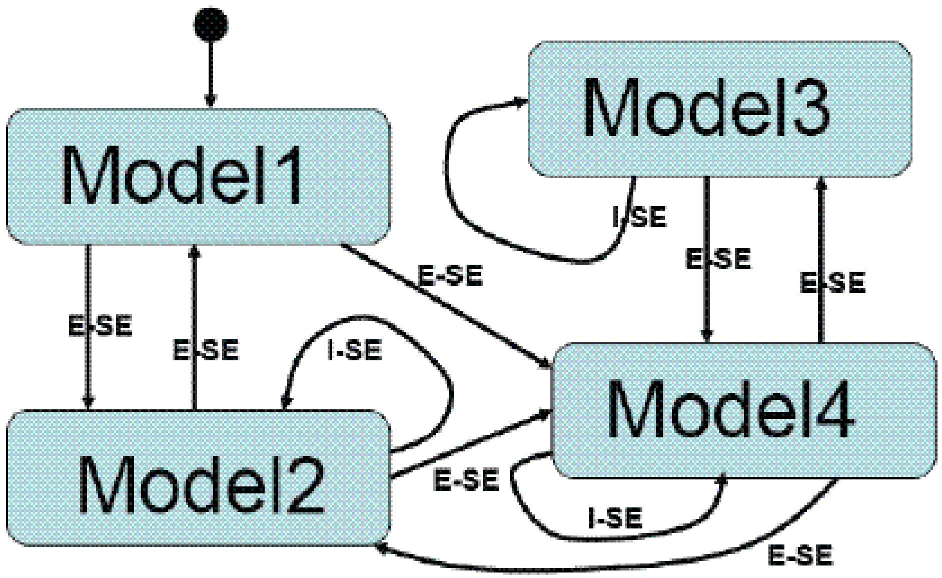


Benchmark Structural-dynamic Systems

3 Case Studies

Bouncing Ball

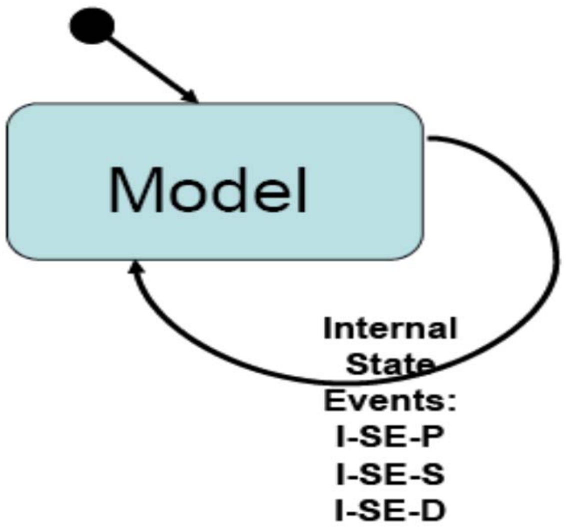




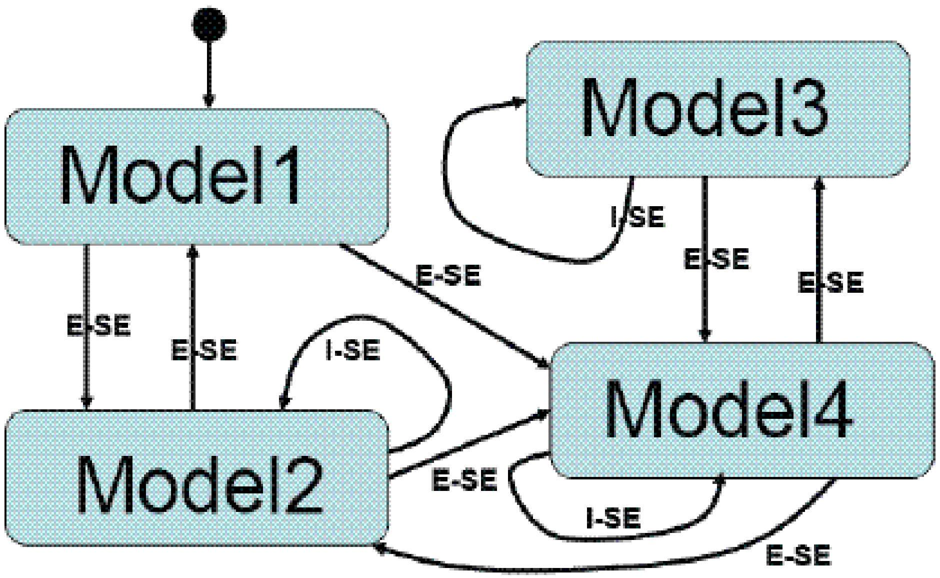
Benchmark Structural-dynamic Systems

3 Case Studies

Bouncing Ball



RLC Circuit with Diode

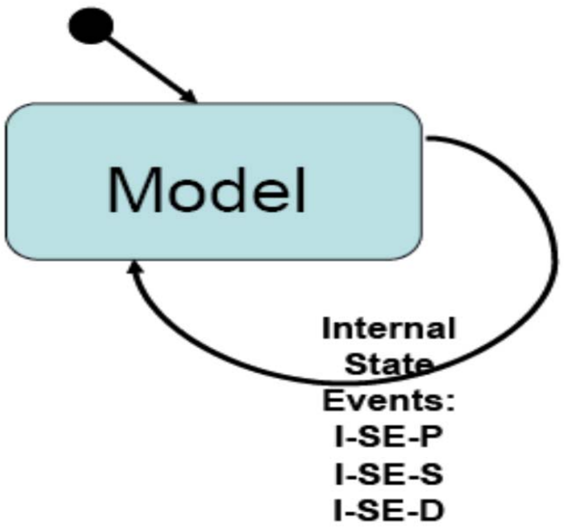


ASC TU VIENNA Mathematical Modelling and Simulation

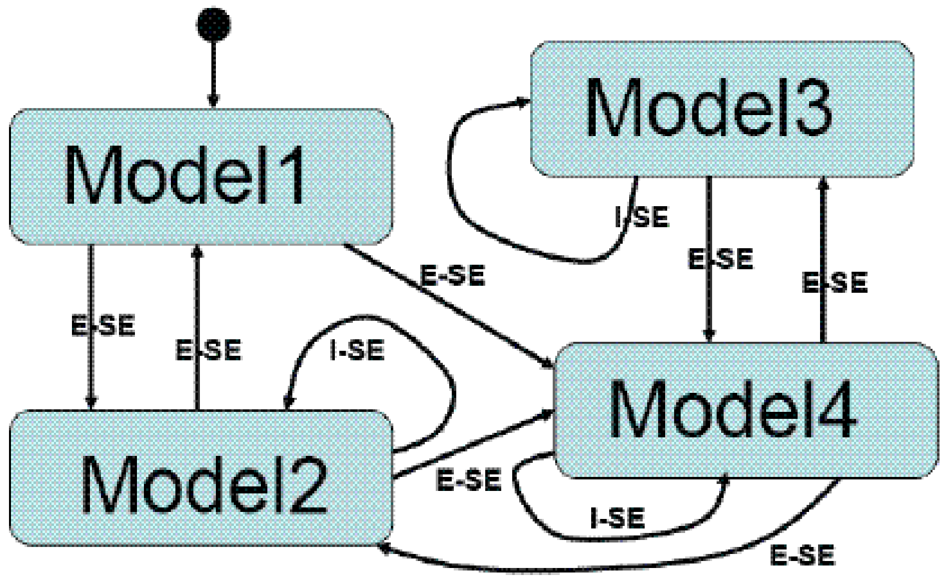
Benchmark Structural-dynamic Systems

3 Case Studies

Bouncing Ball



RLC Circuit with Diode



Rotating Pendulum with Free Flight Phase

Benchmark Structural-dynamic Systems

Bouncing Ball

ASC TU VIENNA Mathematical Modelling and Simulation

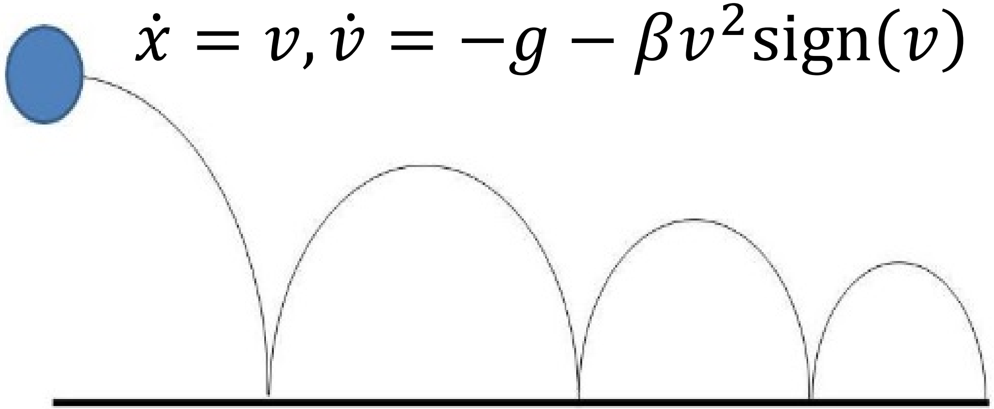
Benchmark Structural-dynamic Systems

Bouncing Ball



Bouncing Ball Model
- Event Contact

$$\dot{x} = v, \dot{v} = -g - \beta v^2 \mathrm{sign}(v)$$



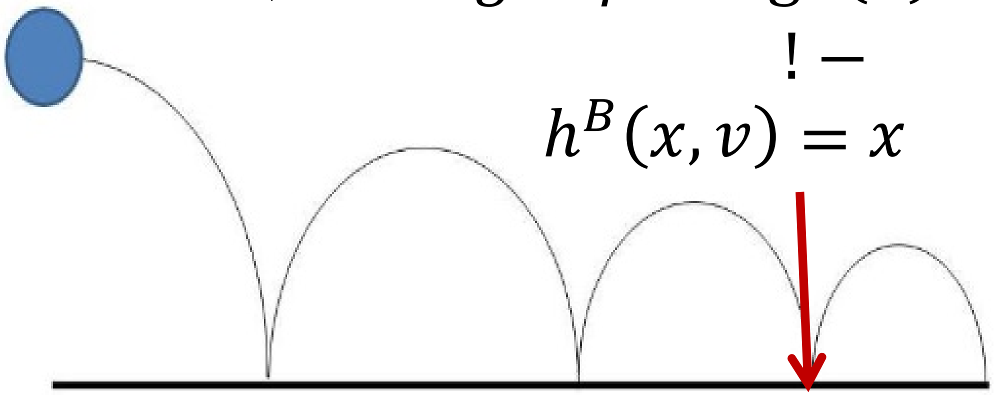
Benchmark Structural-dynamic Systems

Bouncing Ball

Bouncing Ball Model
- Event Contact

$$\dot{x} = v, \dot{v} = -g - \beta v^2 \mathrm{sign}(v)$$

$$h^B(x, v) = x \overset{!}{=} 0$$



$$v_{new}(\hat{t}) \overset{\hat{t}}{\rightarrow} -\mu \cdot v_{prev}(\hat{t}).$$

ASC TU VIENNA Mathematical Modelling and Simulation

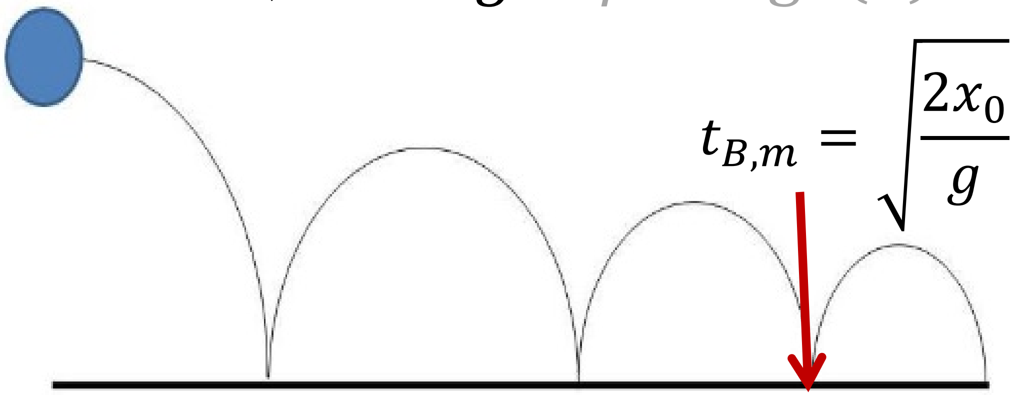
Benchmark Structural-dynamic Systems

Bouncing Ball

Bouncing Ball Model
- Event Contact

$$\dot{x} = v, \dot{v} = -g - \beta v^2 \mathrm{sign}(v)$$

$$t_{B,m} = \sqrt{\frac{2x_0}{g}} \cdot \left(-1 + 2\sum_{i=0}^{m-1} \mu^i\right)$$



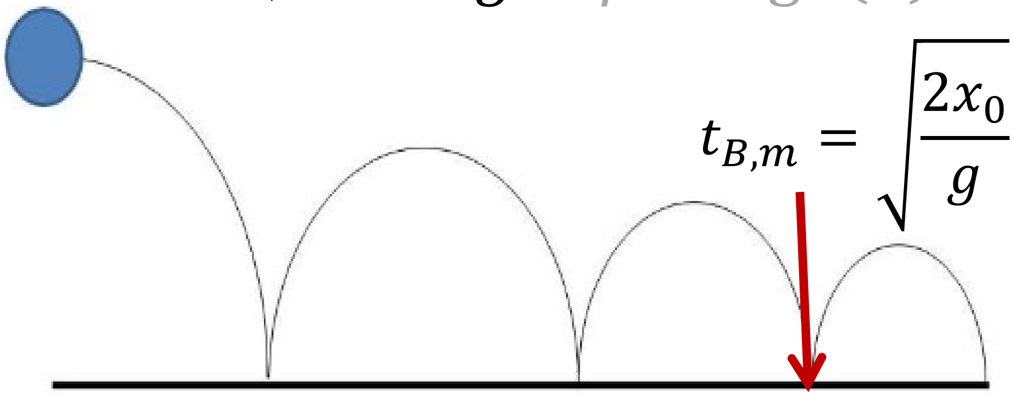
ASC TU VIENNA Mathematical Modelling and Simulation

Benchmark Structural-dynamic Systems

Bouncing Ball

Bouncing Ball Model
- Event Contact

$$\dot{x} = v, \dot{v} = -g - \beta v^2 \mathrm{sign}(v)$$



$$t_{B,m} = \sqrt{\frac{2x_0}{g}} \cdot \left(-1 + 2\sum_{i=0}^{m-1} \mu^i\right)$$

**Infinite # Bounces-
Finite Time !**

$$t_{\mathrm{B},\infty} = \sqrt{\frac{2x_0}{g}} \cdot \frac{1+\mu}{1-\mu}$$

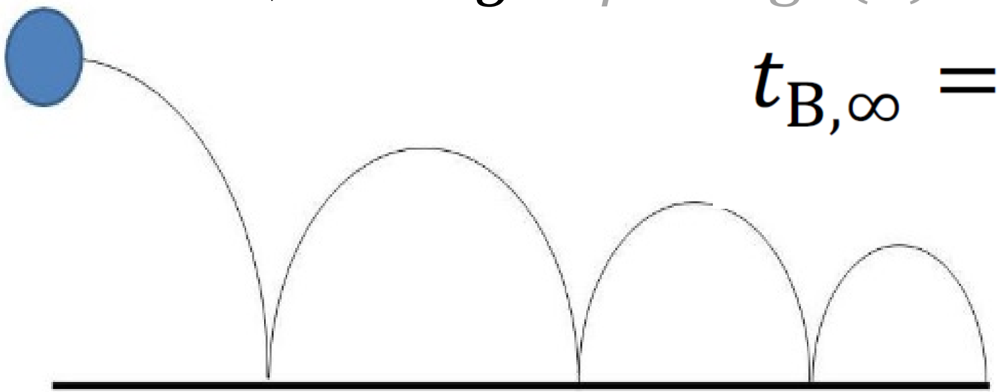
ASC TU VIENNA Mathematical Modelling and Simulation

Benchmark Structural-dynamic Systems

Bouncing Ball

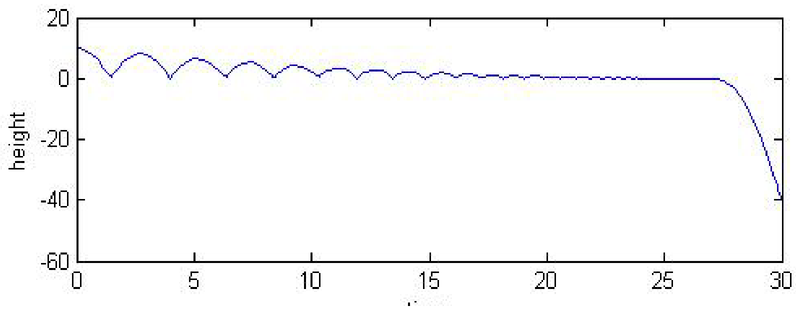
Bouncing Ball Model
- Event Contact

$$\dot{x} = v, \dot{v} = -g - \beta v^2 \text{sign}(v)$$



$$t_{B,\infty} = \sqrt{\frac{2x_0}{g} \cdot \frac{1+\mu}{1-\mu}}$$

**Infinite # Bounces-
Finite Time**



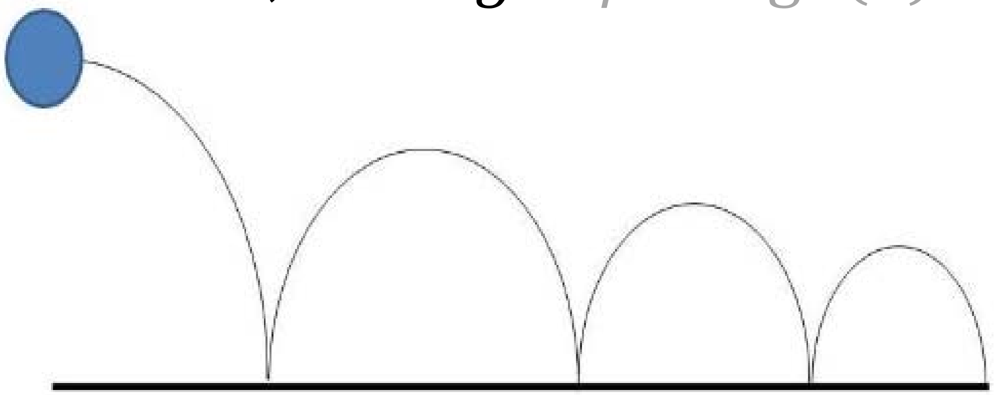
ASC TU VIENNA **Mathematical Modelling and Simulation**

Benchmark Structural-dynamic Systems

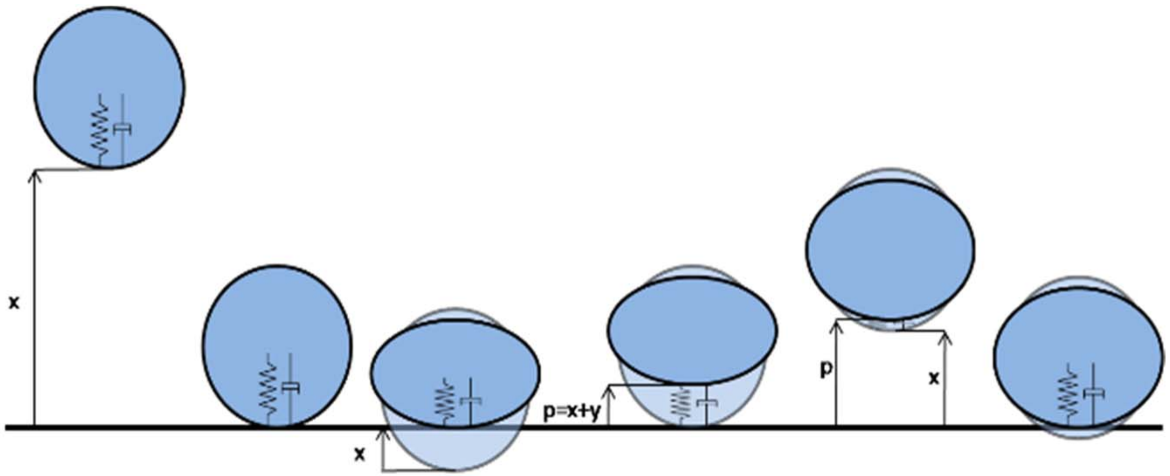
Bouncing Ball

Bouncing Ball Model - Event Contact

$$\dot{x} = v, \dot{v} = -g - \beta v^2 \mathrm{sign}(v)$$



Bouncing Ball Model - Dynamic Contact



Benchmark Structural-dynamic Systems

Bouncing Ball

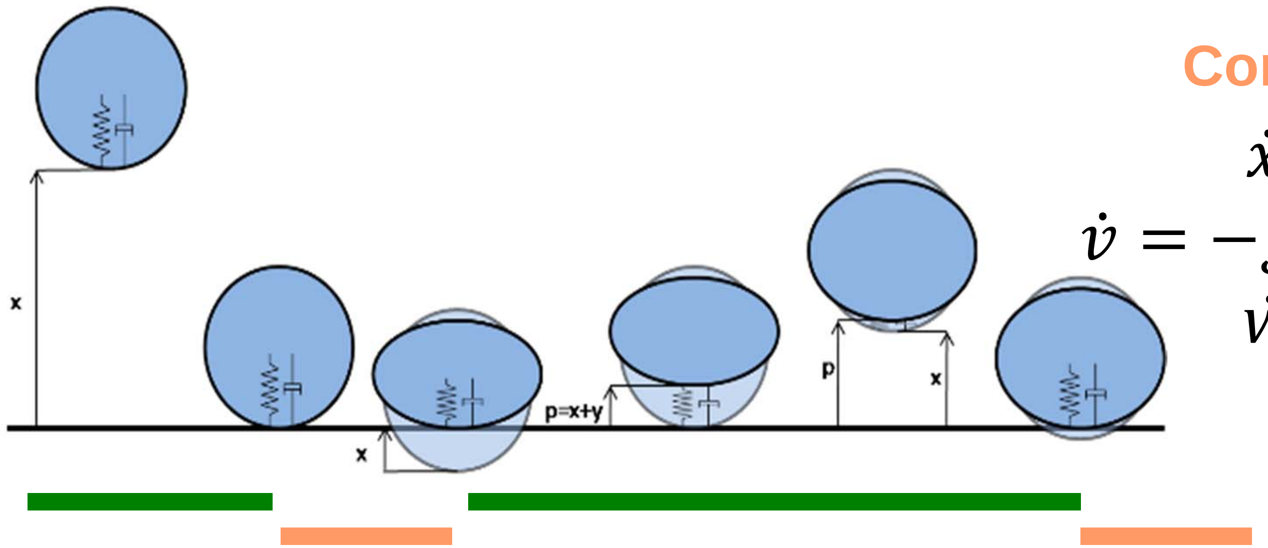
Flying Phase

$$\dot{x} = v$$
$$\dot{v} = -g - \beta v^2 \text{sign}(v)$$
$$\dot{w} = -\frac{k}{d} \cdot w$$

Bouncing Ball Model - Dynamic Contact

Contact Phase

$$\dot{x} = v,$$
$$\dot{v} = -g - kx - dv$$
$$\dot{w} = -v$$



ASC TU VIENNA Mathematical Modelling and Simulation

Benchmark Structural-dynamic Systems

Bouncing Ball



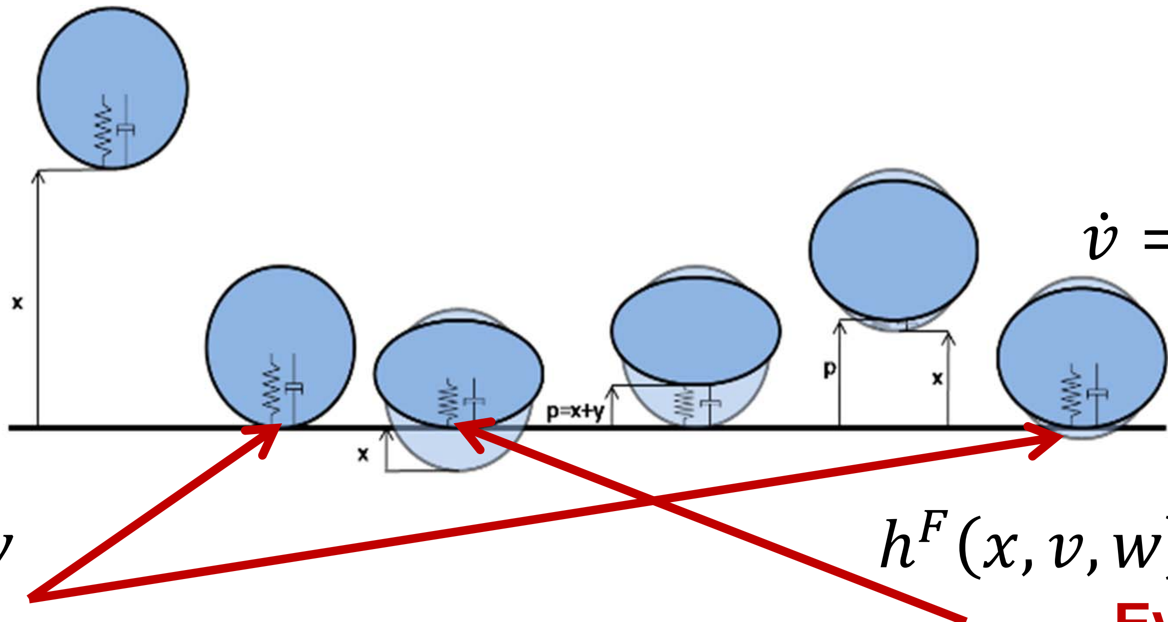
Flying Phase

$$\dot{x} = v$$
$$\dot{v} = -g - \beta v^2 \operatorname{sign}(v)$$
$$\dot{w} = -\frac{k}{d} \cdot w$$

Bouncing Ball Model - Dynamic Contact

Contact Phase

$$\dot{x} = v,$$
$$\dot{v} = -g - kx - dv$$
$$\dot{w} = -v$$



! –

$$h^C(x,v,w) = x + w$$

Event *Contact*

! –

$$h^F(x,v,w) = -kx - dv$$

Event Fly *Restart*

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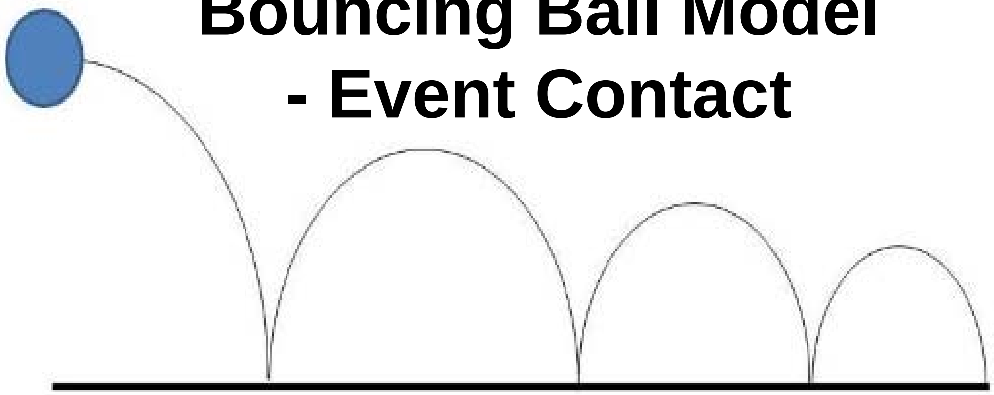
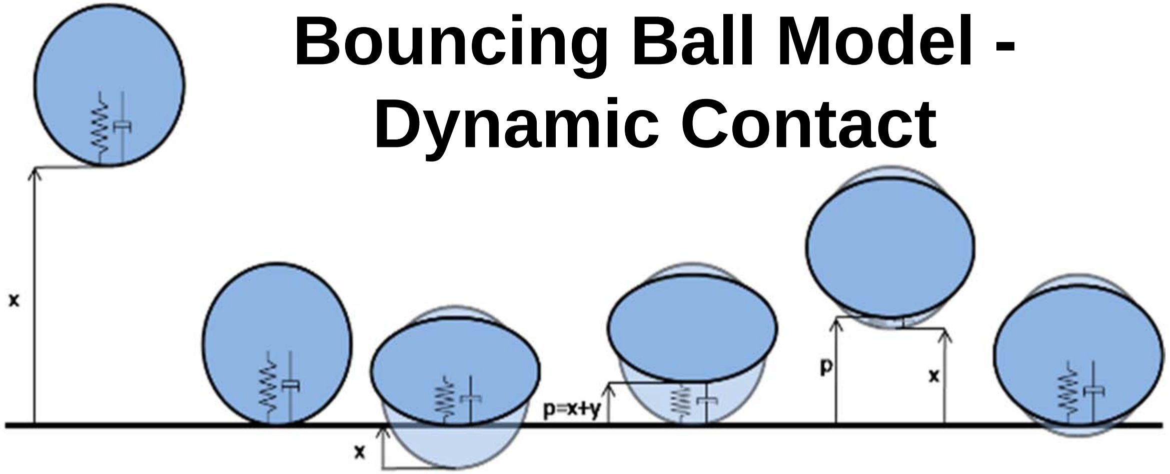
Benchmark Structural-dynamic Systems

Bouncing Ball - Tasks

Bouncing Ball Model - Event Contact



Bouncing Ball Model - Dynamic Contact



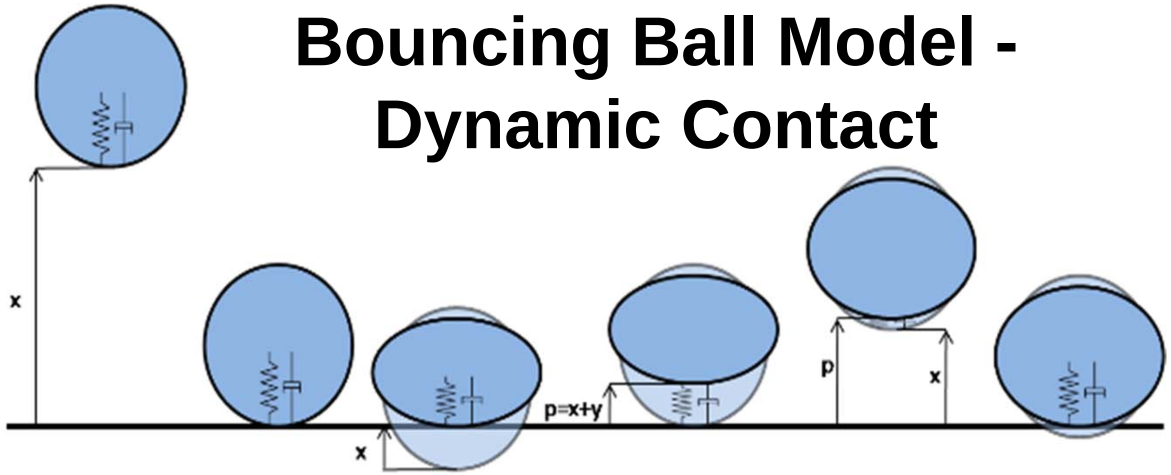
- Description of model implementation
- Simulation until last bounce – scattering prevention
- Testing accuracy of event handling
- Compensation of linear model deviation. .

- Description of model implementation
- Dependency of results from algorithms.
- Investigation of contact phase
- Parameter studies.
-

Benchmark Structural-dynamic Systems

Bouncing Ball - Tasks

Bouncing Ball Model - Dynamic Contact



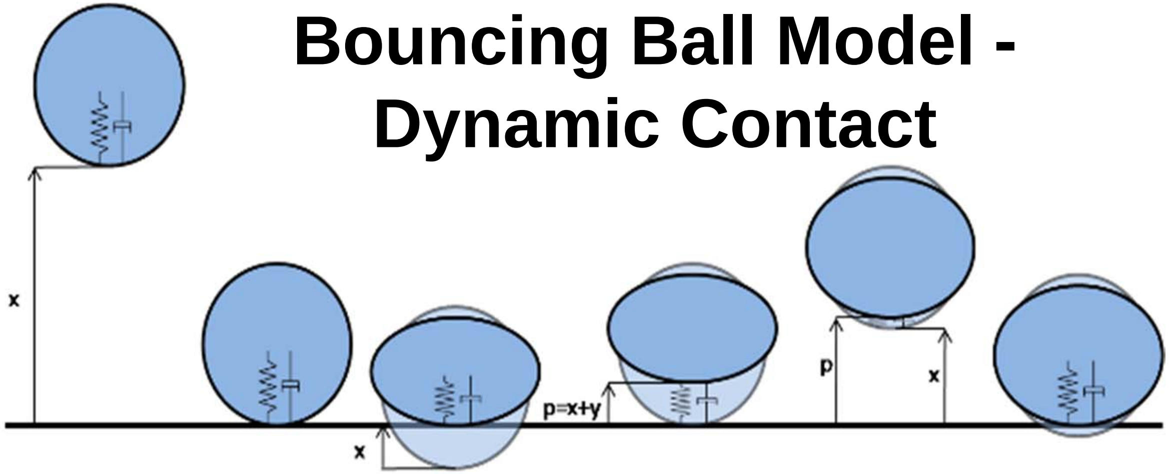
- Description of model implementation
- Dependency of results from algorithms.
- Investigation of contact phase
- Parameter studies.
- **Bouncing ball on Mars.**

Benchmark Structural-dynamic Systems

Bouncing Ball - Tasks

- *In a publication of NASA, there was a speculation of a big inflatable ball, which can be used to damp down the impact of a lander at atmospheric entry.*
- *http://saturn.astrobio.net/pressrelease/63/*

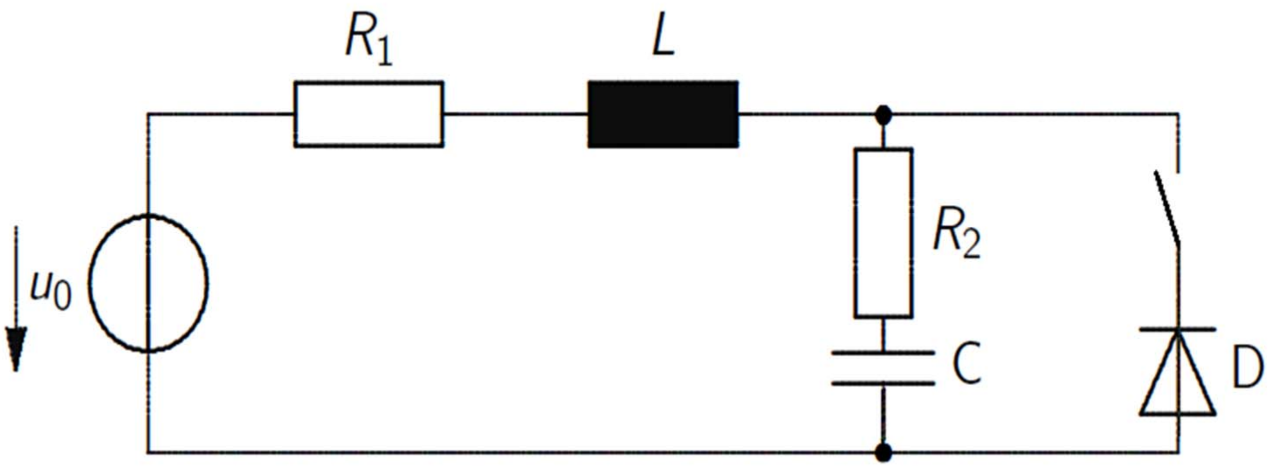
Bouncing Ball Model - Dynamic Contact



- Description of model implementation
- Dependency of results from algorithms.
- Investigation of contact phase
- Parameter studies.
- **Bouncing ball on Mars.**

Benchmark Structural-dynamic Systems

RLC Circuit with Diode



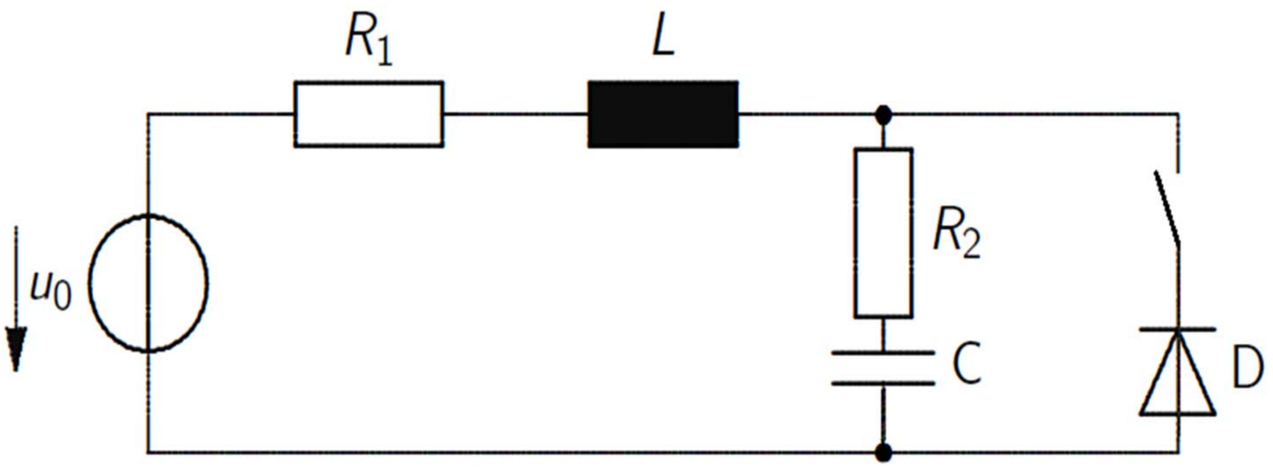
$$u_C + u_{R_1} + u_L + u_{R_2} + u_C = u_0$$

$$u_{R_2} + u_C = u_D$$

$$\frac{di}{dt} = \frac{1}{L} u_L, \qquad \frac{du_C}{dt} = \frac{1}{C}(i + i_D)$$

Benchmark Structural-dynamic Systems

RLC Circuit with Diode



Diode Model

$$u_D = \hat{F}(i_D)$$

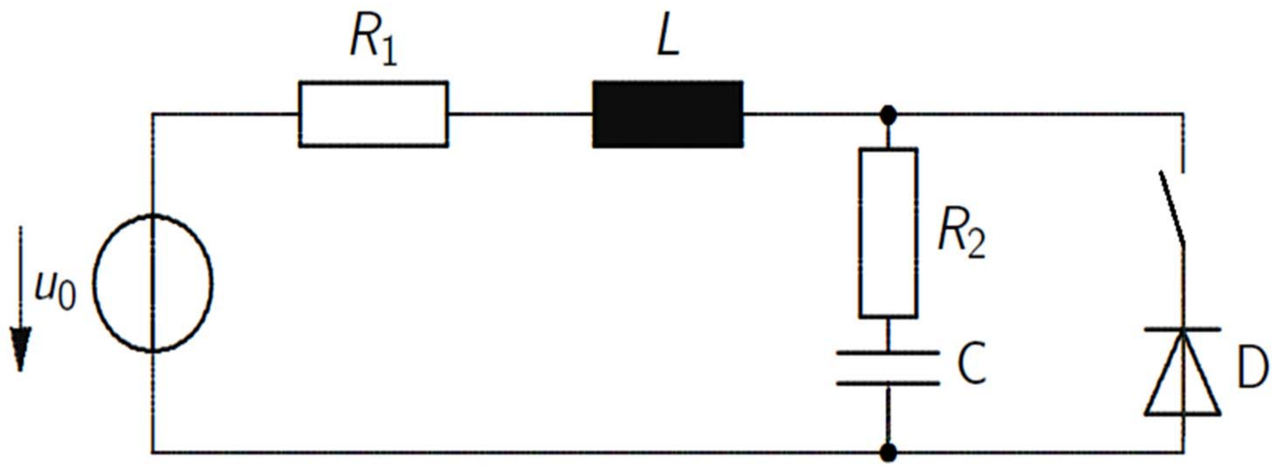
$$u_C + u_{R_1} + u_L + u_{R_2} + u_C = u_0$$

$$u_{R_2} + u_C = u_D$$

$$\frac{di}{dt} = \frac{1}{L} u_L, \qquad \frac{du_C}{dt} = \frac{1}{C}(i + i_D)$$

Benchmark Structural-dynamic Systems

RLC Circuit with Diode



Diode Model

$$u_D = \hat{F}(i_D)$$

Locking Phase

$$i_D = 0 \ if \ \ u_D < 0$$

Conducting Phase

$$u_D > 0$$

$$0 = R_2(i + i_D) + u_C - F(i_D)$$

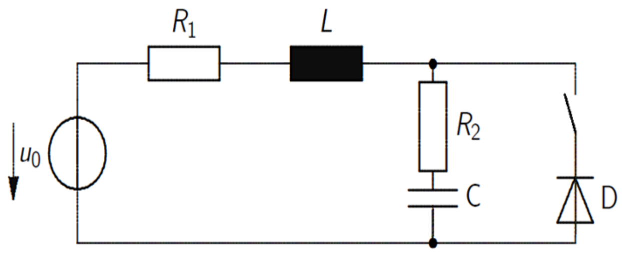
$$u_C + u_{R_1} + u_L + u_{R_2} + u_C = u_0$$

$$u_{R_2} + u_C = u_D$$

$$\frac{di}{dt} = \frac{1}{L} u_L, \qquad \frac{du_C}{dt} = \frac{1}{C}(i + i_D)$$

Benchmark Structural-dynamic Systems

RLC Circuit with Diode



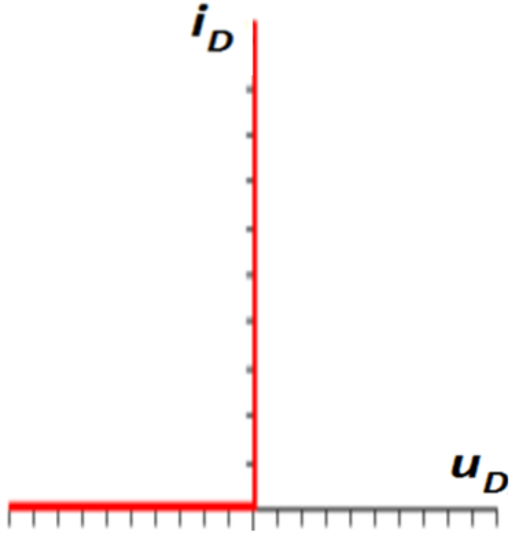
Diode Model $$u_D = \hat{F}\left(i_D\right)$$

Locking Phase

$$i_D = 0 \;\; if \;\; u_D < 0$$

Conducting Phase

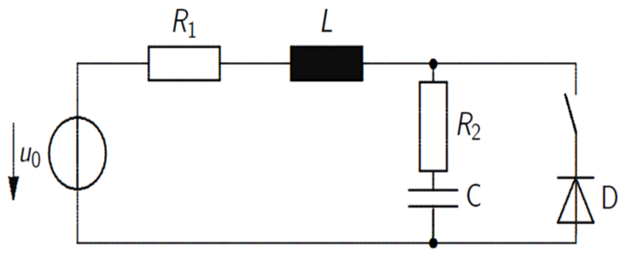
$$u_D > 0 \quad 0 = R_2(i + i_D) + u_C - F(i_D)$$



Ideal Switch

Benchmark Structural-dynamic Systems

RLC Circuit with Diode



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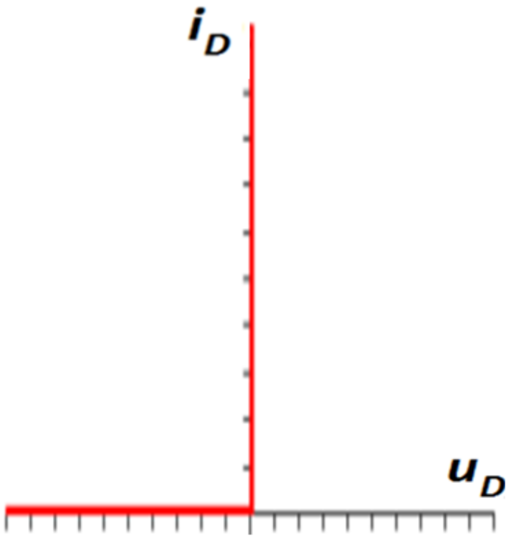

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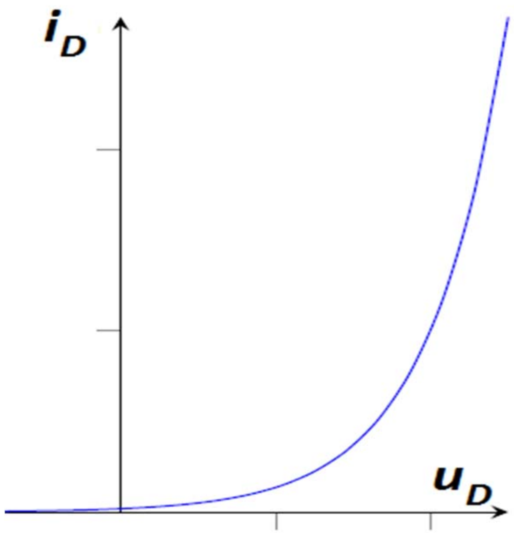
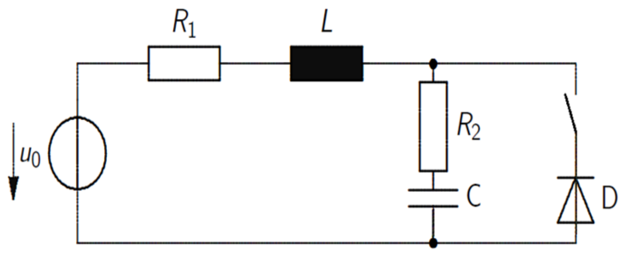


Shockley Function

Benchmark Structural-dynamic Systems

RLC Circuit with Diode



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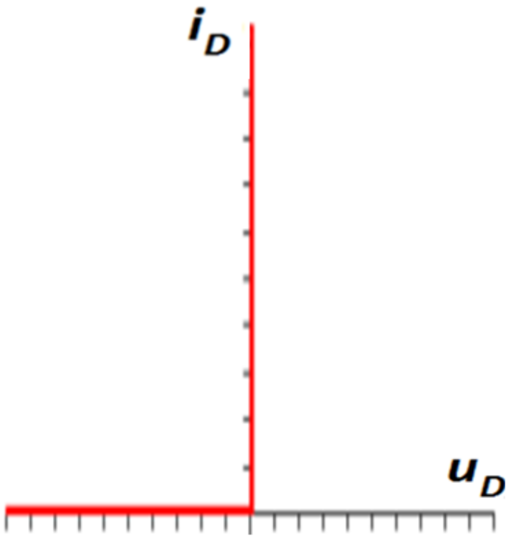

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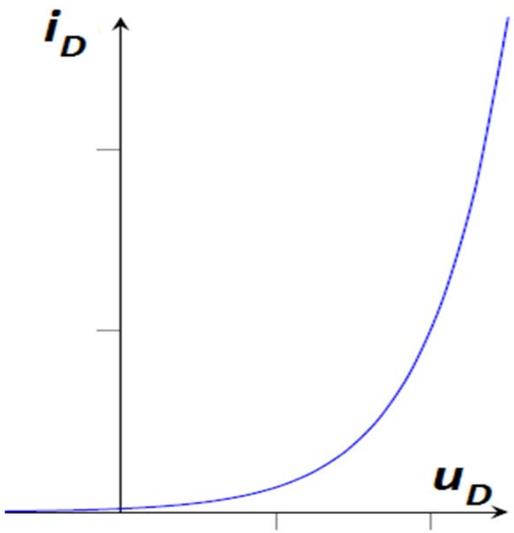

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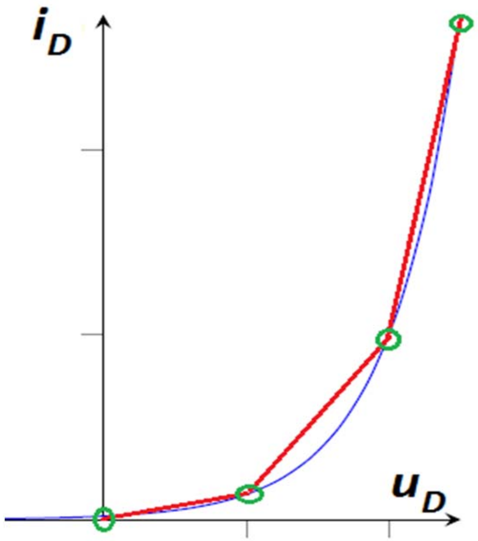
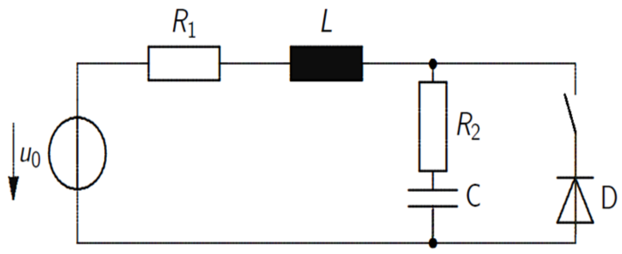


Shockley Charact. Curve

Benchmark Structural-dynamic Systems

RLC Circuit with Diode



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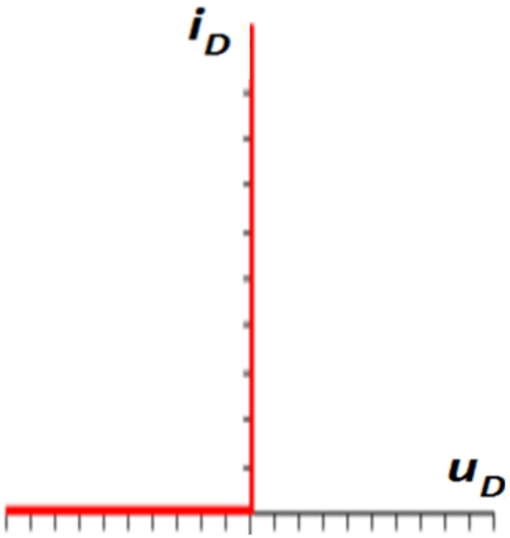

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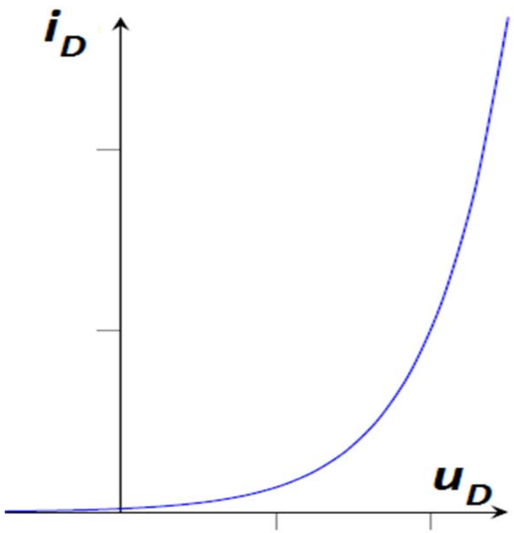

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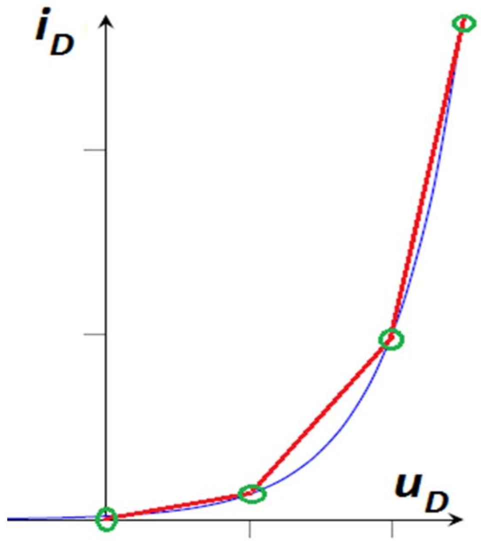
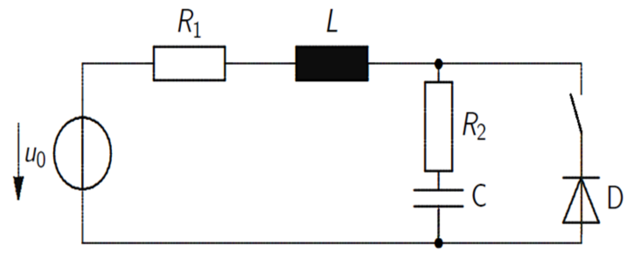


Shockley Charact. Curve

Benchmark Structural-dynamic Systems

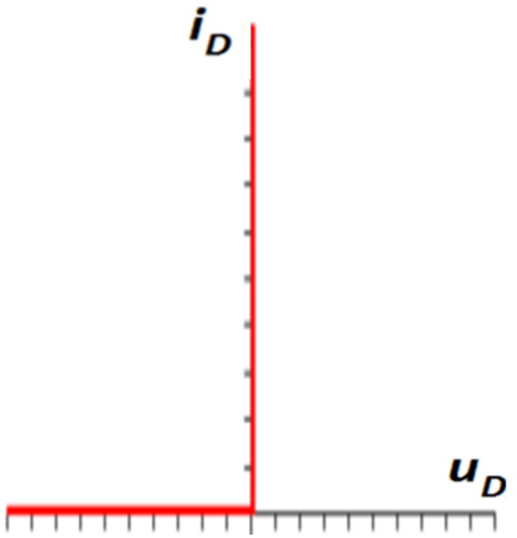
RLC Circuit with Diode



Diode Model $$u_D = \hat{F}\,(i_D)$$

Conducting Phase $u_D > 0$

$$\frac{du_C}{dt} = \frac{1}{R_2} u_C, \frac{di}{dt} = -\frac{R_1}{L} i + \frac{1}{L} u_0$$
$$u_D = R_2 i + u_C$$



Ideal Switch

Locking Phase $u_D < 0$

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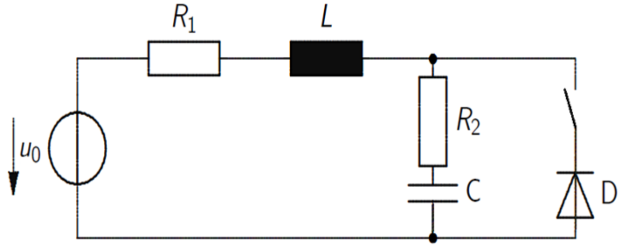
RLC Circuit with Diode



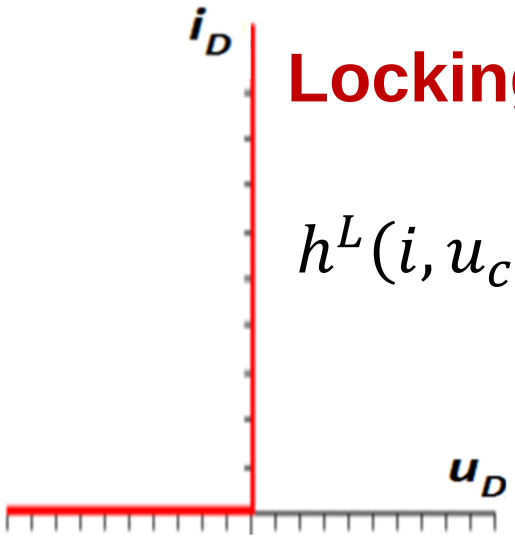
Diode Model $u_D = \hat{F}\,(i_D)$

Conducting Phase Start

$$h^C(i,u_c) = u_D \overset{!}{=} R_2 i + u_C \quad (! +)$$

Locking Phase Start

$$h^L(i,u_c,i_D) = u_D \overset{!}{=} R_2(i+i_D) + u_C \quad (! -)$$



Ideal Switch

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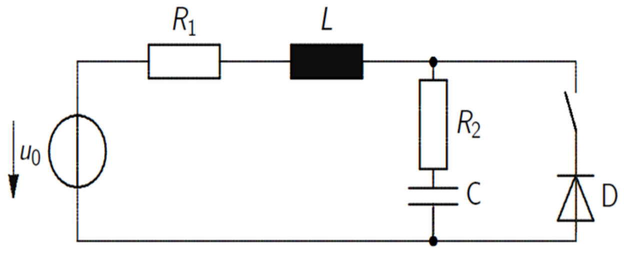
Benchmark Structural-dynamic Systems

RLC Circuit with Diode



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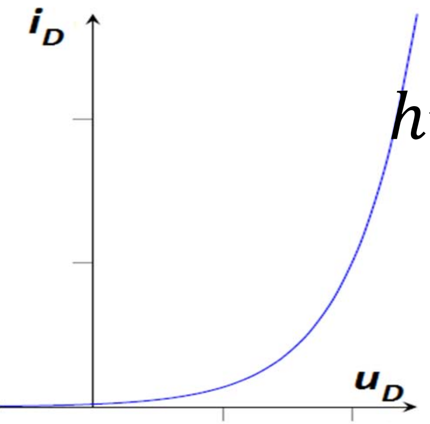

Shockley Function

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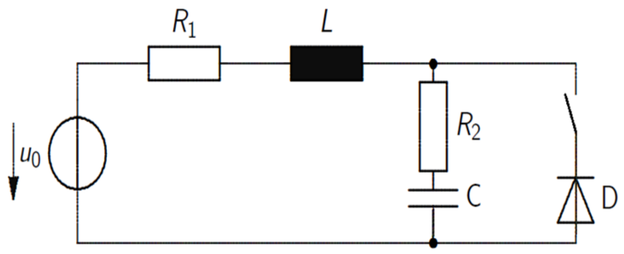
RLC Circuit with Diode



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Conducting Phase

$u_D > 0$

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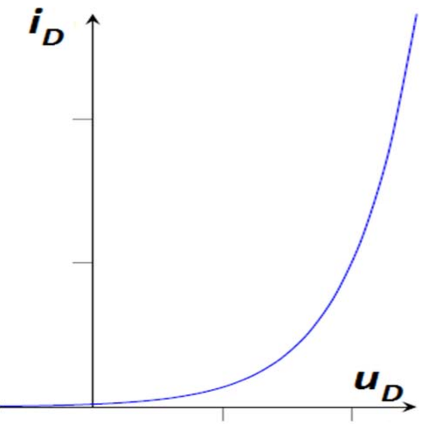
$$I_S R_2 \left(e^{\frac{u_d}{U_T}} - 1\right) + u_c + u_d = 0$$



Shockley Function

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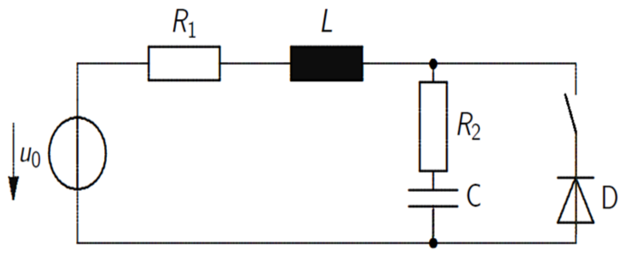

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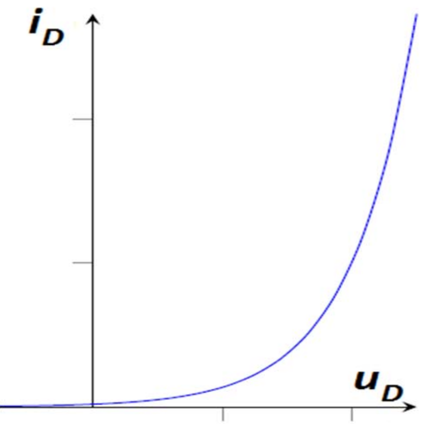
$$R_2(i + i_D) + u_C - U_T \ln\left(\frac{i_D}{I_S} + 1\right) = 0$$
$$u_d = U_T \ln\left(\frac{i_D}{I_S} - 1\right)$$



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Benchmark Structural-dynamic Systems

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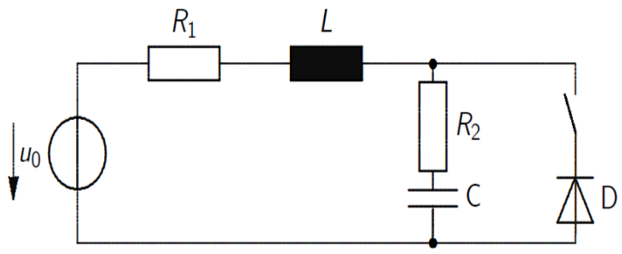

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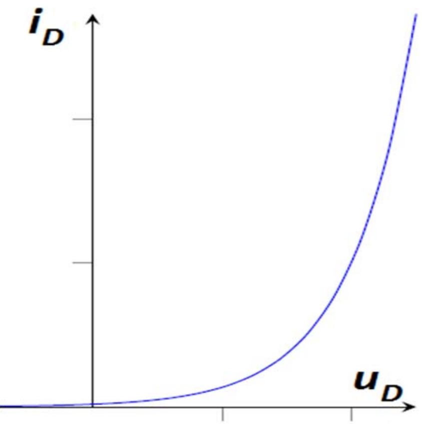
$$\dot{u}_D = -\dot{u}_C \left(I_S R_2 \left(e^{\frac{u_D}{U_T}}\right) \frac{1}{U_T} + 1 \right)^{-1}$$



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Shockley Function

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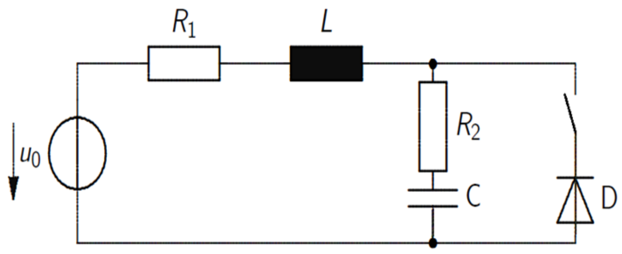

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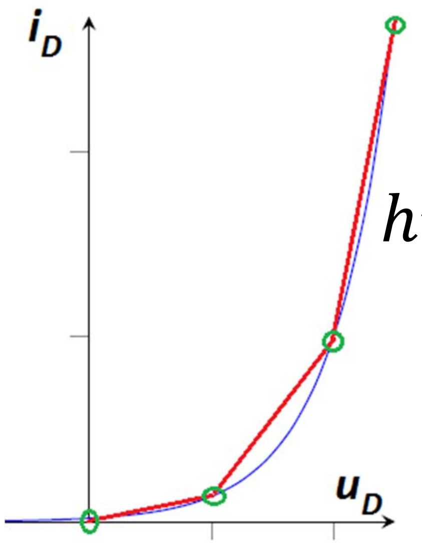

Shockley Charact. Curve

Benchmark Structural-dynamic Systems

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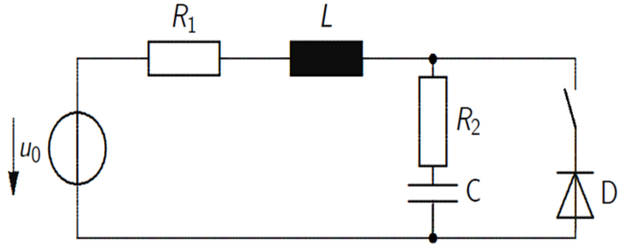

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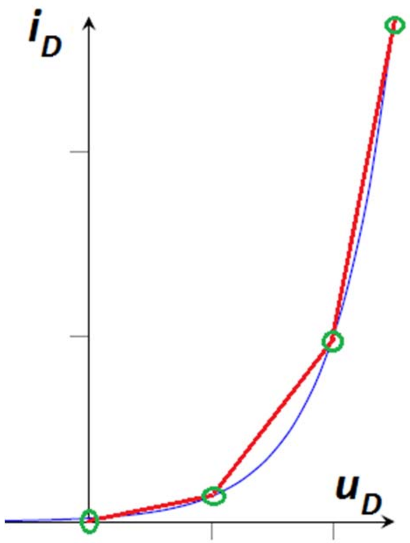

Shockley Charact. Curve

Benchmark Structural-dynamic Systems

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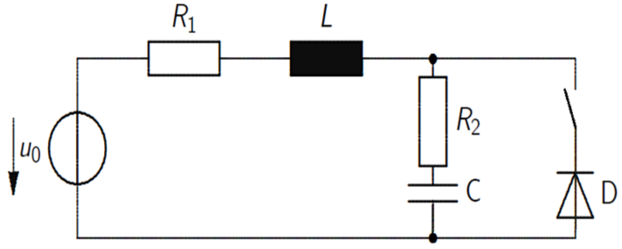

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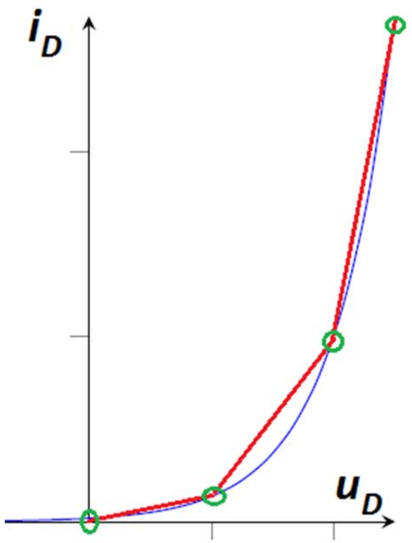
$$F_{LIN,j}(u_c(t); (u_{D,j}, i_{D,j}) + u_c + u_d = 0$$



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Shockley Charact. Curve

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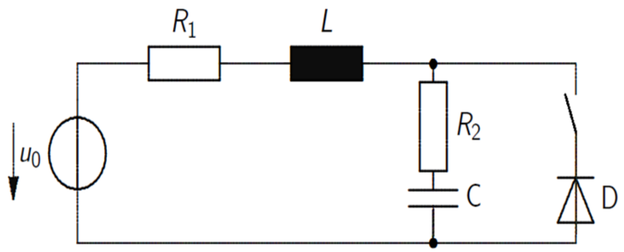
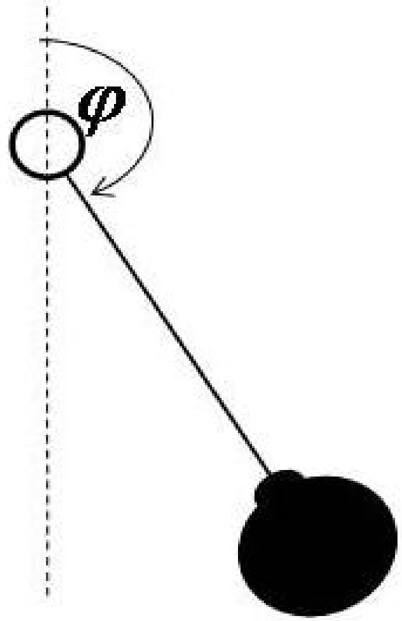
RLC Circuit with Diode - Tasks

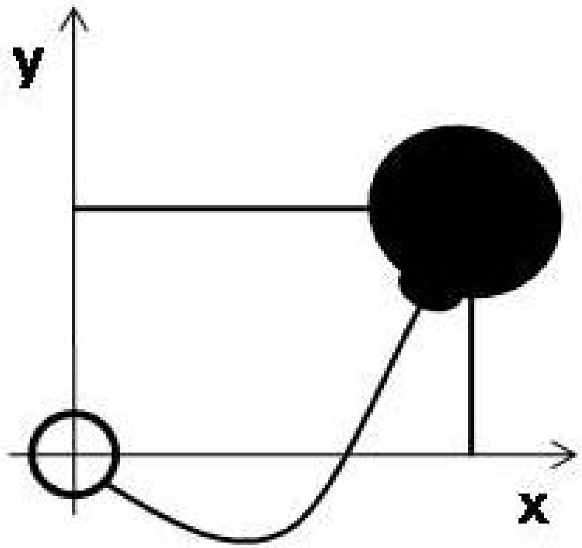


- Description of model implementations.
- Dependency of results from algorithms. (Shortcut vs Shockley)
- Comparison of Shortcut and Shockley diode model.
- Approximation of Shockley diode model (Shockley vs. Char. Curve)
- Relevance of choice of algebraic state (i_d vs u_d)
- Investigation for real-time simulation index reduction).

Benchmark Structural-dynamic Systems

Rotating Pendulum with Free Flight Phase

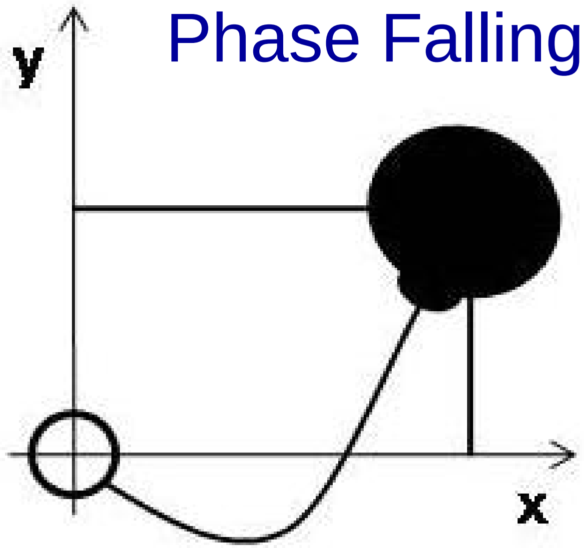




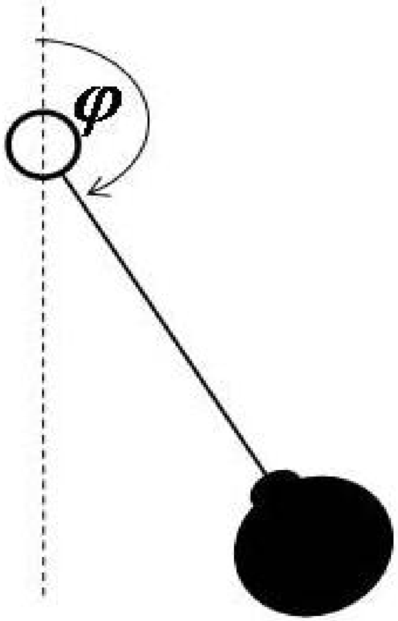
Benchmark Structural-dynamic Systems

Rotating Pendulum with Free Flight Phase

Phase Falling



$$m\ddot{x} = -k\dot{x},$$
$$m\ddot{y} = -mg - k\dot{y}$$



$$\ddot{\varphi} + \frac{k}{m}\dot{\varphi} - \frac{g}{l}\sin(\varphi) = 0$$

Phase Swinging

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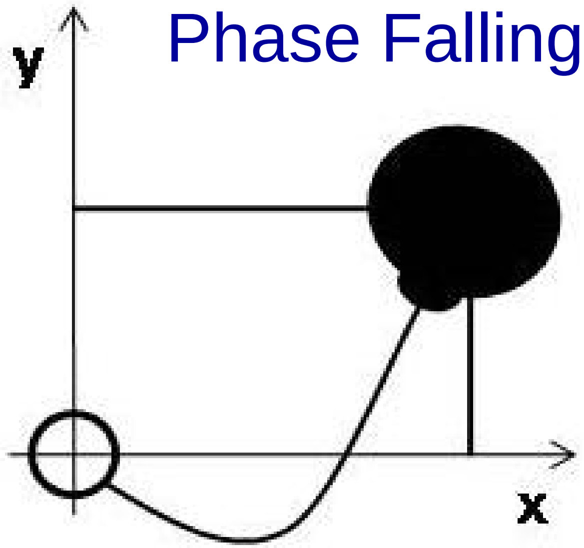
Rotating Pendulum with Free Flight Phase

Phase Falling



$$m\ddot{x} = -k\dot{x},$$
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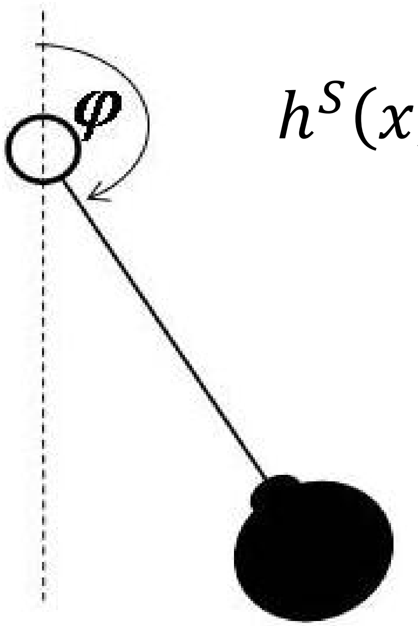
Event Start *Swinging*

! +

$$h^S(x,y) = d^2 - l^2 = x^2 + y^2 - l^2$$



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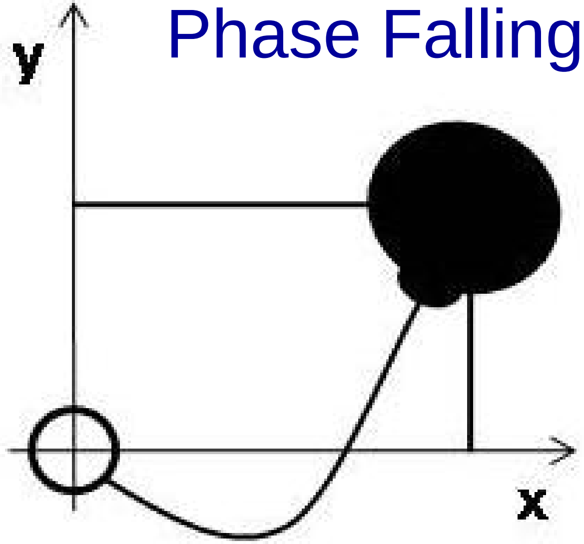
Phase Swinging

Benchmark Structural-dynamic Systems

Rotating Pendulum with Free Flight Phase

Phase Falling



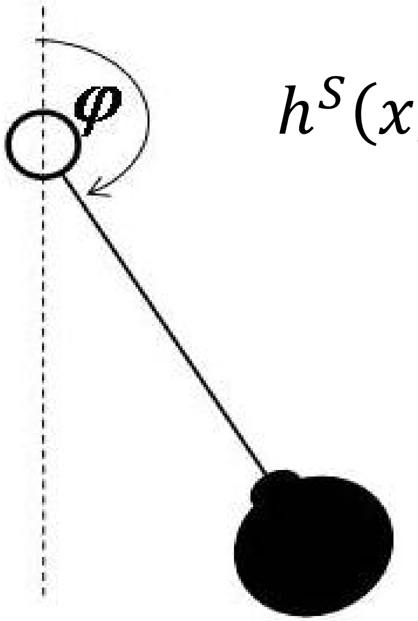
$$m\ddot{x} = -k\dot{x},$$
$$m\ddot{y} = -mg - k\dot{y}$$

Event *Start Swinging*

$$h^S(x,y) \overset{!\,+}{=} d^2 - l^2 = x^2 + y^2 - l^2$$

Event *Start Falling*

$$h^F(\varphi,\dot{\varphi}) \overset{!\,-}{=} F = -gm\,cos(\varphi) + ml\dot{\varphi}^2$$



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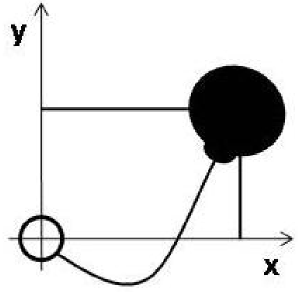
Phase Swinging

Benchmark Structural-dynamic Systems

Rotating Pendulum with Free Flight Phase



$F < 0\,?$

$\varphi, \dot{\varphi} \rightarrow x, \dot{x}, y, \dot{y}$

$\varphi, \dot{\varphi}, F$

circular movement

(rope tight)

$$\ddot{\varphi} = \frac{g}{l}\sin(\varphi) - \frac{k}{m}\dot{\varphi}$$

$$F = -mg\cos(\varphi) + ml\dot{\varphi}^2$$

free fall

(rope loose)

$$m\ddot{x} = -k\dot{x}$$

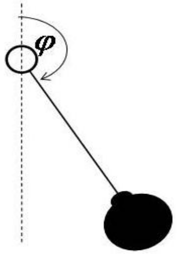
$$m\ddot{y} = -mg - k\dot{y}$$

$$r^2 = x^2 + y^2$$

$x, \dot{x}, y, \dot{y}, r$

$x, \dot{x}, y, \dot{y} \rightarrow \varphi, \dot{\varphi}$

$r \geq l\,?$

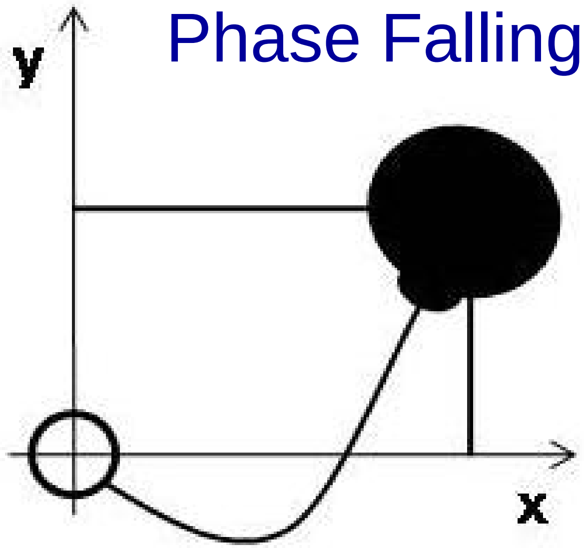


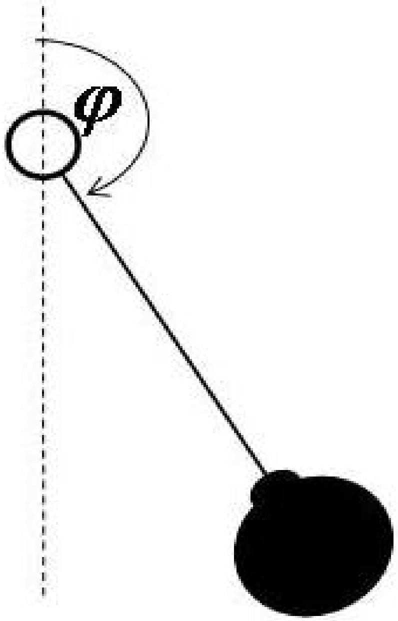
Benchmark Structural-dynamic Systems

Rotating Pendulum with Free Flight Phase

$$m\ddot{x} = -k\dot{x},$$
$$m\ddot{y} = -mg - k\dot{y}$$

Phase Falling





Phase Swinging

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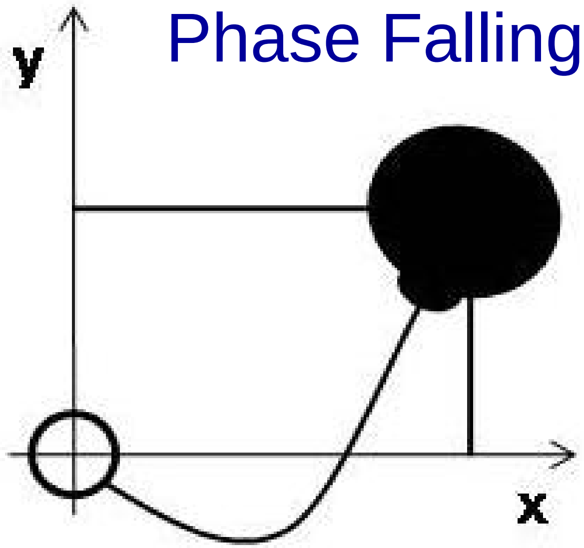
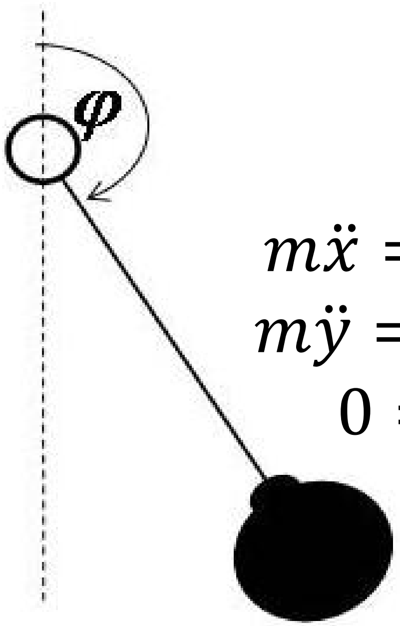
Rotating Pendulum with Free Flight Phase

Phase Falling

$$m\ddot{x} = -k\dot{x},$$
$$m\ddot{y} = -mg - k\dot{y}$$





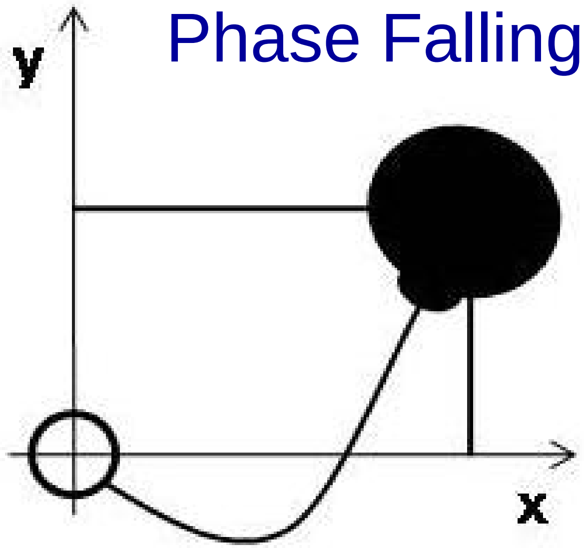
$$m\ddot{x} = -k\dot{x} - \lambda x$$
$$m\ddot{y} = -mg - k\dot{y} - \lambda x$$
$$0 = x^2 + y^2 - l^2$$

Phase Swinging

Benchmark Structural-dynamic Systems

Rotating Pendulum with Free Flight Phase

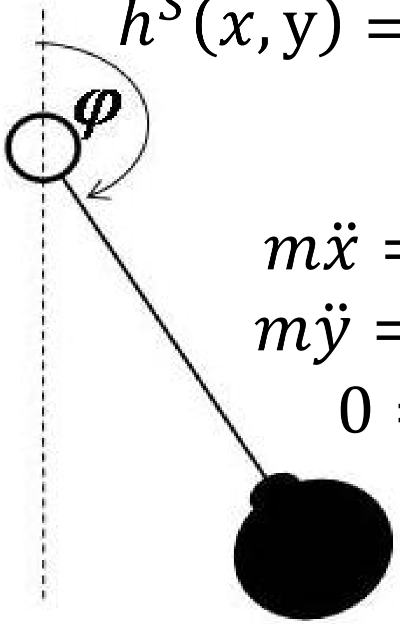
Phase Falling



$$m\ddot{x} = -k\dot{x},$$
$$m\ddot{y} = -mg - k\dot{y}$$

Event *Start Swinging*

$$h^S(x,y) \overset{!\,+}{=} d^2 - l^2 = x^2 + y^2 - l^2$$



$$m\ddot{x} = -k\dot{x} - \lambda x$$
$$m\ddot{y} = -mg - k\dot{y} - \lambda x$$
$$0 = x^2 + y^2 - l^2$$

Event *Start Falling*

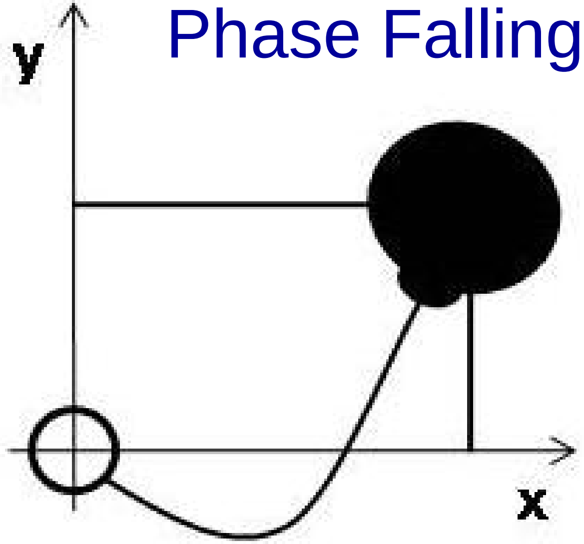
$$h^F(x,\dot{x},y,\dot{y}) \overset{!\,-}{=} F = -gm\,\frac{y}{l} + ml \cdot f(x,\dot{x},y,\dot{y})$$

Phase Swinging

Benchmark Structural-dynamic Systems

Rotating Pendulum with Free Flight Phase

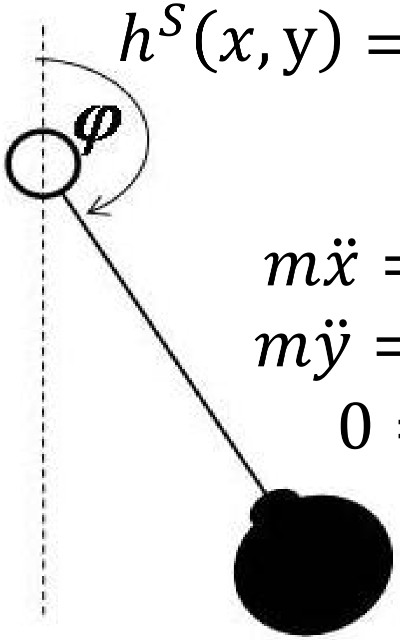
Phase Falling



$$m\ddot{x} = -k\dot{x},$$
$$m\ddot{y} = -mg - k\dot{y}$$

Event *Start Swinging*

$$h^S(x,y) \overset{!+}{=} d^2 - l^2 = x^2 + y^2 - l^2$$



$$m\ddot{x} = -k\dot{x} - \lambda x$$
$$m\ddot{y} = -mg - k\dot{y} - \lambda x$$
$$0 = x^2 + y^2 - l^2$$

DAE index 3

Event *Start Falling*

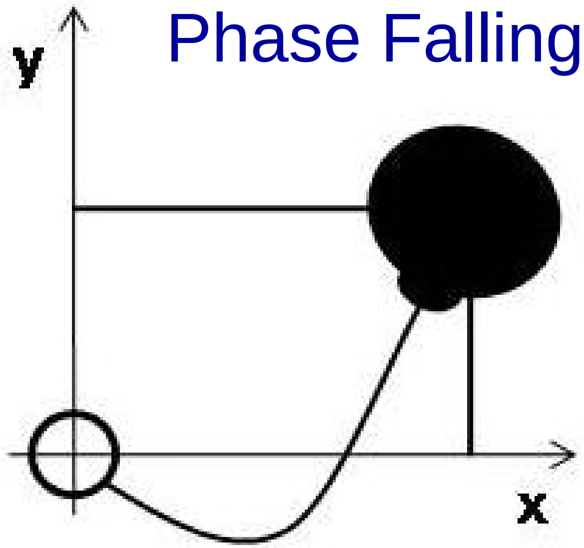
$$h^F(x,\dot{x},y,\dot{y}) \overset{!-}{=} F = -gm\,\frac{y}{l} + ml \cdot f(x,\dot{x},y,\dot{y})$$

Phase Swinging

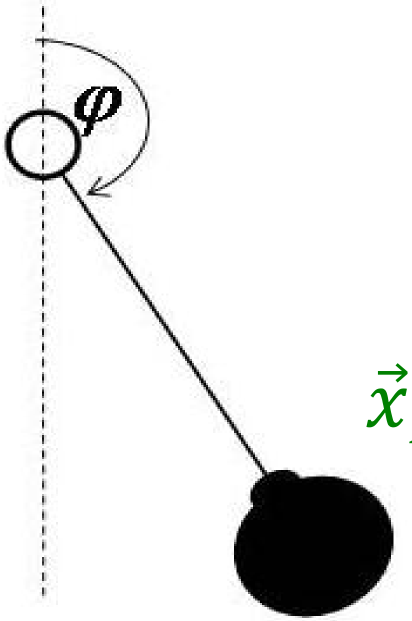
Benchmark Structural-dynamic Systems

Rotating Pendulum with Free Flight Phase

Phase Falling



$$\vec{x}_S(t) = \left(\varphi(t), \dot{\varphi}(t)\right)^T$$



$$\vec{x}_F(t) = \left(x(t), \dot{x}(t), y(t), \dot{y}(t)\right)^T$$

$$\vec{x}_M(t) = \left(\varphi(t), \dot{\varphi}(t), x(t), \dot{x}(t), y(t), \dot{y}(t)\right)^T$$

$$\vec{x}\,(t) = \left(x(t), \dot{x}(t), y(t), \dot{y}(t)\right)^T$$

Phase Swinging

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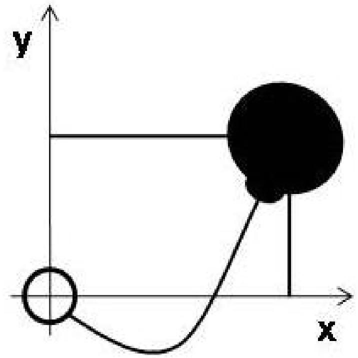
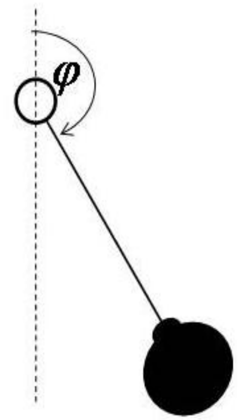
Rotating Pendulum with Free Flight Phase - Tasks



- Description of model implementations (state pace, ODEs, physical modelling, ….).
- Basic simulation of phases (parameter studies
- Dependency of results from algorithms (check event handling)
- External energy supply (kick pendulum dependent on state)



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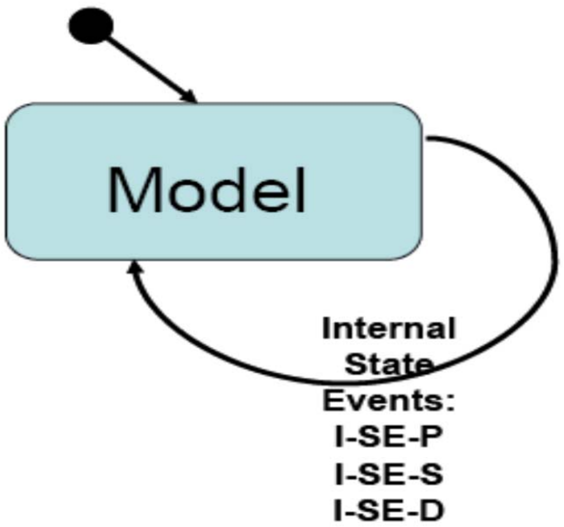
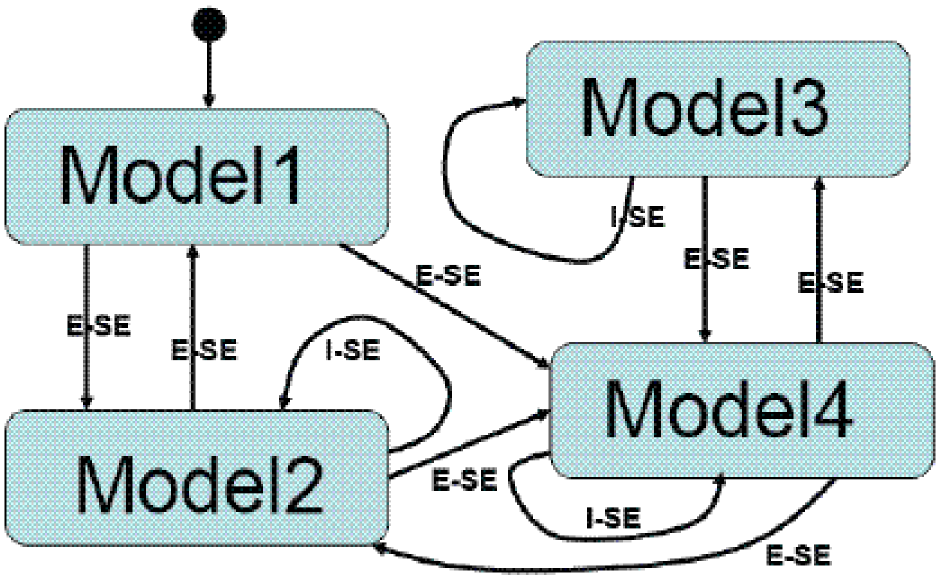
3 Case Studies

Bouncing Ball



RLC Circuit with Diode



Rotating Pendulum with Free Flight Phase

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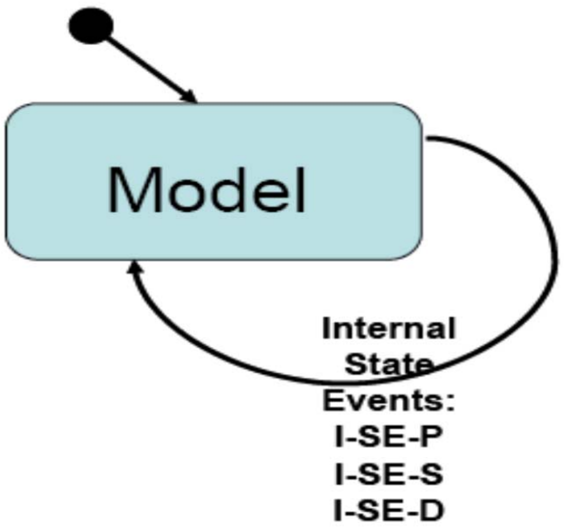
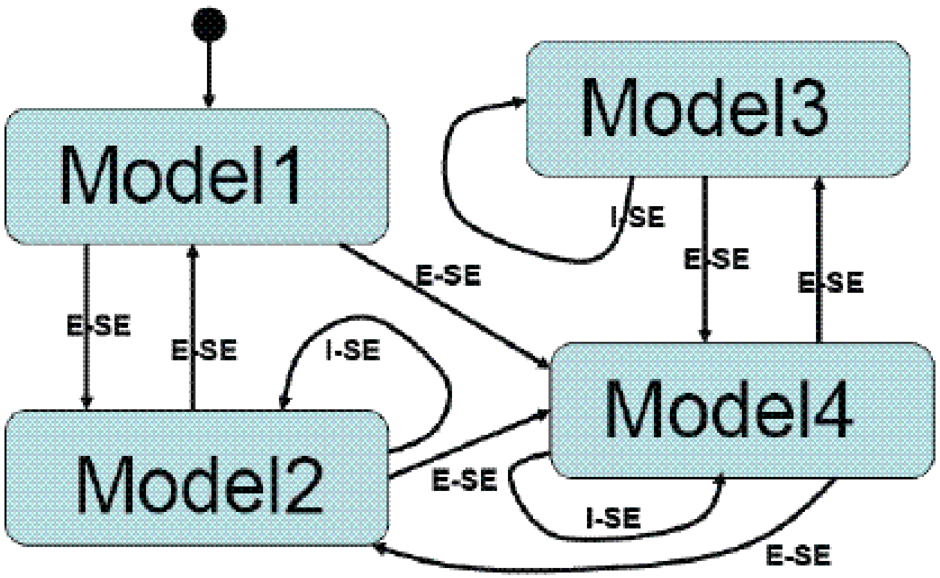


RLC Circuit with Diode



Rotating Pendulum with Free Flight Phase

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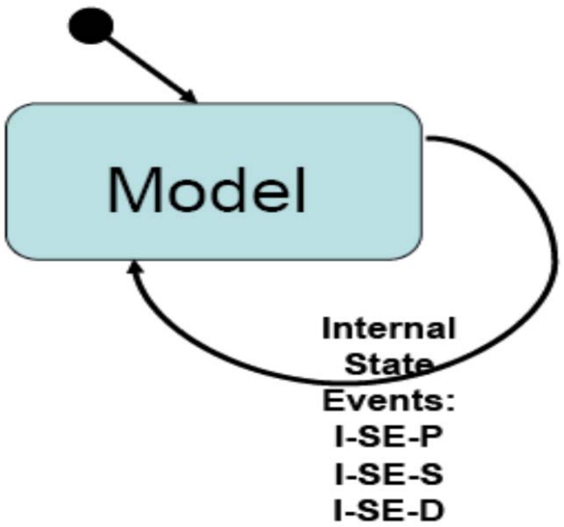
- **Solution documentation and publication in SNE of 'solutions' may take more pages – up to 10 pages SNE.**

Bouncing Ball



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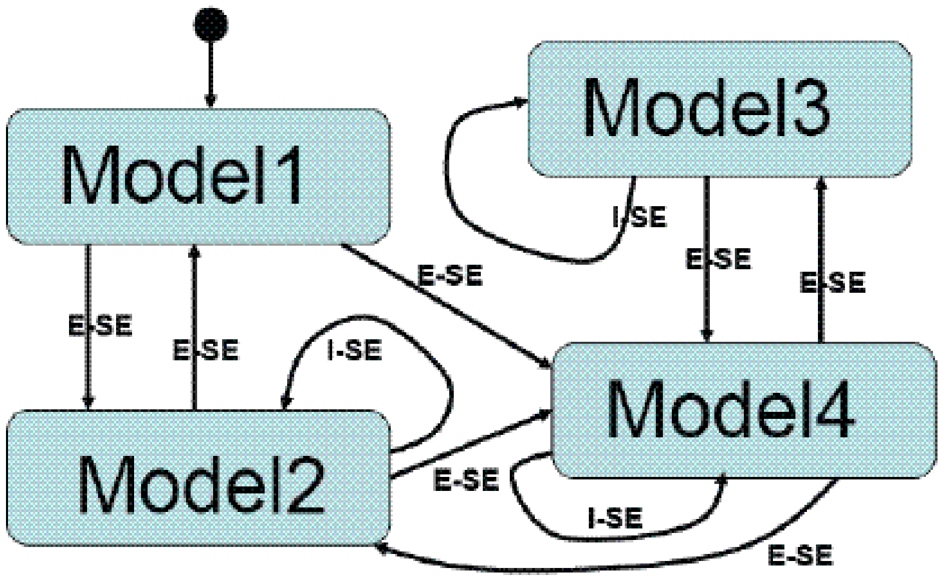

Rotating Pendulum with Free Flight Phase

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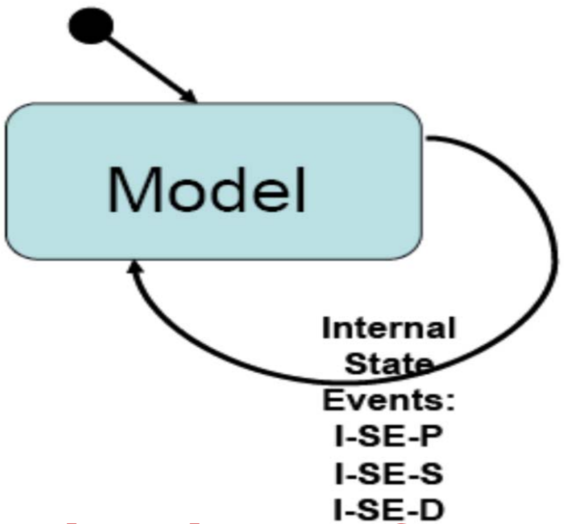
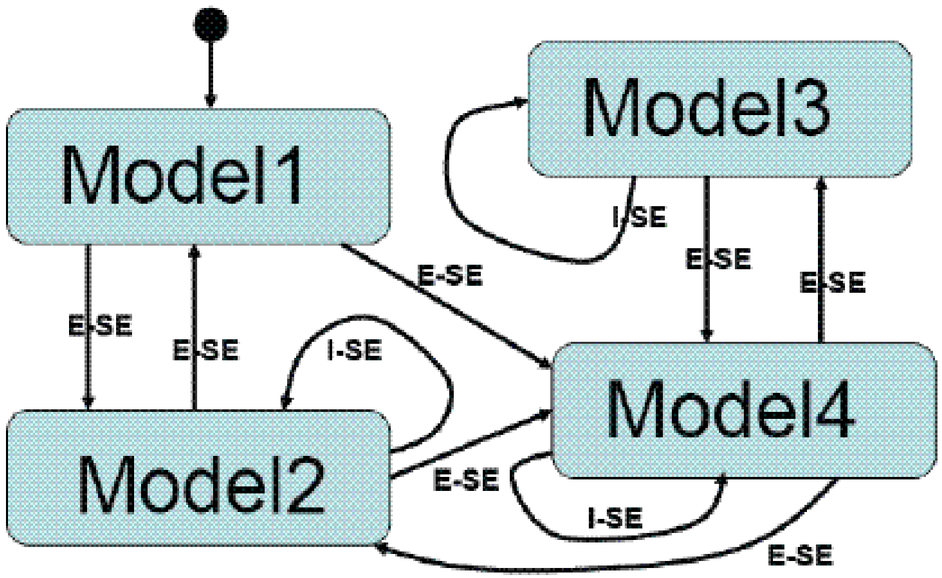


RLC Circuit with Diode



Rotating Pendulum with Free Flight Phase

- **Thank you for Attention !**
- **Happy Benchmarking !**
- **Happy Writing !**
- **Waiting for your Submission !**